Universität des Saarlandes
FR 6.2 Informatik

Antonios Antoniadis and Martin Hoefer **SS 2014**

Excercises
Online Algorithms

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss14/OnlineAlgos/

Sheet 3 Deadline: 29.05.2014

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam.

Exercise 1 (*10 points*)
Searching on a star is a generalization of the cow-path problem: There are m rays all originating from the same point s. An agent starts at s and is searching for some item located at some unknown distance on an (again unknown) ray. We consider the algorithm that visits the rays in a circular fashion, and walks a distance of

$$x_i = \left(\frac{m}{m-1}\right)^i$$

on the i'th ray it visits. Prove that the above algorithm, has a competitive ratio of

$$1 + 2\left(\frac{m^m}{(m-1)^{m-1}}\right).$$

Exercise 2 (*15 points*)
We consider the problem of online scheduling with the objective of minimizing the total flow time $\sum_i (C_i - r_i)$ of the schedule. Algorithm *rank round robin (RRR)* schedules at every instantaneous time t each unfinished task i for an amount proportional to $rank_t(i)$, where $rank_t(i)$ denotes the number of tasks that are unfinished at t and were released no later than r_i, i.e., $rank_t(i) := |\{j | r_j \leq r_i \text{ \& } j \text{ is unfinished at } t\}|$.
So, if there are k unfinished tasks at timepoint t, the i'th of them (ordered by release times) would be assigned an $i / \sum_{j=1}^{k} j$ fraction of the processor.
Prove that RRR is $(2+\epsilon)$-speed $O(1)$-competitive.

Hint: Use the potential function

$$\Phi(t) := \sum_{i \in RRR(t)} z_i(t) rank_t(j),$$

where (as in the lecture), $z_i(t) = \max\{p_i^{RRR}(t) - p_i^{OPT}(t), 0\}$*,* $p_i^A(t)$ *is the processing volume that is unfinished for task* i *under algorithm A at timepoint* t*, and* $RRR(t)$ *is the set of unfinished tasks under RRR at time* t*.*

Exercise 3 (*8+7 points*)
Recall the online scheduling problem for minimizing the total flow time $\sum_i (C_i - r_i)$ on a single-processor from the lecture. The algorithm *Shortest Remaining Processing Time (SRPT)* processes at every point in time the task with the shortest remaining processing time among all unfinished tasks. Prove that:

a) SRPT achieves a competitive ratio of 1 for the objective of minimizing total flow-time on a single processor.
Hint: Try to use proof by contradiction, and apply an exchange argument.

b) SRPT has a competitive ratio strictly greater than 1 for the objective of weighted flow time on a single processor. In this problem every task i is also associated with a weight w_i and we wish to minimize

$$\sum_i w_i \left(C_i - r_i\right).$$