Universität des Saarlandes
FR 6.2 Informatik

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Excercises
Online Algorithms

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss14/OnlineAlgos/

Sheet 4 Deadline: 12.06.2014

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam.

Exercise 1 (*5+5 points*)
Consider the following variation of the graph exploration problem seen in the lecture: Starting from a vertex s, we wish to explore all the *vertices* of strongly connected, directed graph $G = (V, E)$, and the cost incured is the number of edges it traversed during the exploration. Let

$$c = \frac{n}{2} + \frac{1}{2} - \frac{1}{n}.$$

- Prove that no deterministic online algorithm can obtain a competitive ratio better than c.
- Develop and analyze a deterministic online algorithm that has a ratio of c.

Exercise 2 (*2+2+6 points*) Consider the one-player version of the *memory game*: n pairs of cards lie face down on the table. In each move the player can look at two of the cards. If they are identical they are removed from the table. The player can remember all seen cards and his goal is to remove all cards with the fewest possible moves.

- Give a deterministic online algorithm that takes at mots $2n$ moves to remove all the cards.
- Give a deterministic online algorithm that takes at most $2n - 1$ moves to remove all the cards.
- Show that no deterministic online algorithm can take less than $2n - 1$ moves in the worst case.

Exercise 3 (*4 points*) The integrality gap of a linear program is the ratio of the cost of the best integral solution over the cost of the best fractional solution. Show that the ski rental

LP given in the lecture has integrality gap 1 – i.e., show that the problem has an integral optimum solution that is also optimal for the fractional relaxation.

Exercise 4 (*6+4+6 points*) Consider the following algorithm that solves the fractional ski rental LP in an online fashion (c.f. slide 14 from the lecture). Initially set $z = 0$. Then, for each day j, if $z < 1$ do the following: Set $x_j = 1 - z$; set $z = z(1 + 1/B) + 1/(cB)$, for some $c > 1$; set $y_j = 1$.

- Show that the primal and dual solutions are feasible and on each day, the ratio between the change in the primal and dual objective functions is bounded by $(1 + 1/c)$.
- Explain why these properties imply that the algorithm achieves a competitive ratio of $(1 + 1/c)$ for the fractional ski rental problem.
- Show how to round the output online to obtain a randomized algorithm for the binary ski rental problem with the same competitive ratio.