Notation: e refers to the basis of the natural logarithm, whenever it appears in a mathematical expression.

1. Let $H = (V, E)$ be a k-uniform hypergraph, with m hyperedges.
   (i) Show that if $m < 3^{k-1}$, then H is 3-colorable.
   (ii) Show that if each vertex belongs to $3^{k-1}/(ek)$ many hyperedges, then H is 3-colorable (whatever the total number of hyperedges might be!)

2. Let $G = (V, E)$ be a graph with maximum degree d, and let $V = V_1 \cup V_2 \cup \ldots \cup V_r$ be a partition of V into r pairwise disjoint sets. Suppose each set is of cardinality $|V_i| \geq 2ed$, then there is an independent set of vertices $W \subset V$ such that for each $i = 1, 2, \ldots, r$, $|W \cap V_i| = 1$.

3. Let $G = (V, E)$ be a graph, and suppose each vertex $v \in V$ is associated with a set $S(v)$, of colors, such that $|S(v)| \geq 10d$, where $d \geq 1$. Suppose also, that for each $v \in V$ and $c \in S(v)$, there are at most d neighbours u of v such that $c \in S(u)$. Prove that there is a proper coloring of G (i.e. no two adjacent vertices should have the same color), assigning to each vertex $v \in V$, a color from its class $S(v)$.

4. Prove that for every integer $d > 1$, there is a finite $c = c(d)$ such that the edges of any bipartite graph with maximum degree d, in which every cycle has at least $c(d)$ edges, can be colored by $d + 1$ colors, so that no two adjacent edges have the same color, and no cycle has only two colors.

5. Let $Z_0, Z_1, \ldots$ be a martingale with respect to the sequence $X_0, X_1, \ldots$. Show that $(Z_i)_{i \geq 0}$ is also a martingale sequence with respect to itself.

6. Consider an n-dimensional cube, with $N = 2^n$ nodes. Let S be a non-empty set of vertices on the cube, and let x be a vertex chosen uniformly at random among all vertices of the cube. Let $D(x, S)$ be the minimum number of co-ordinates in which x and y differ over all points $y \in S$. Given $t > 0$, give a bound on
$$Pr[|D(x, S) - E[D(x, S)]| \geq t],$$
in terms of $E[D(x, S)]$, n, and t.