

Beyond classical chip design  
lecture 3

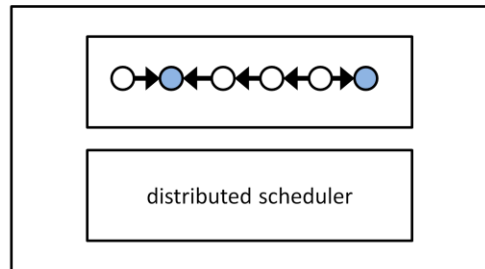
Self-stabilization (continued)

## What we had...

Distributed, weak-fair scheduler ->

Distributed, **neighbour-mutex**, weak fair scheduler.

Dijkstra's algorithm



# Self-stabilization

... link reversal almost solves the problem.

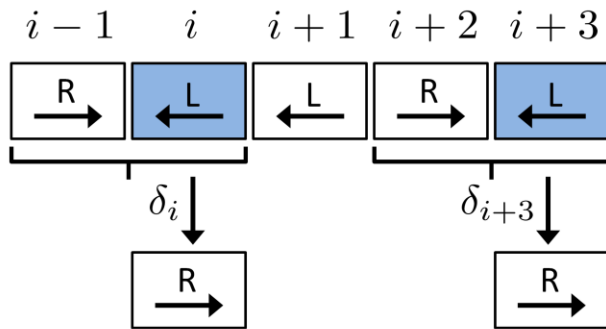
token merging

LR

distributed schedule

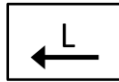
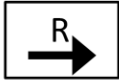
# Self-stabilization

stable algorithm



# Self-stabilization

adding direction



## Self-stabilization

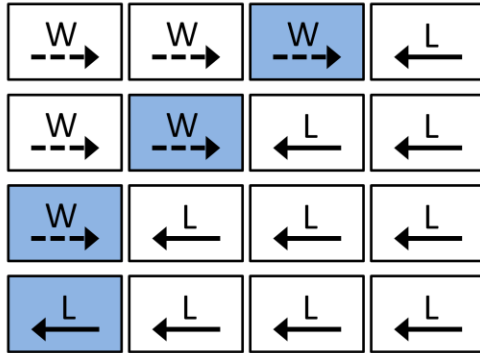
tokens turn only at borders ->

**Prop 1.** Mutex holds.

**Prop 2.** Weak fairness holds.

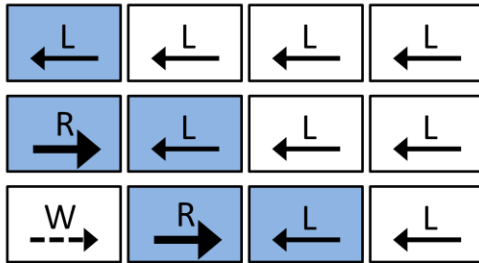
# Self-stabilization

to left ...



# Self-stabilization

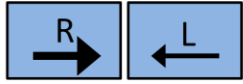
to right ...



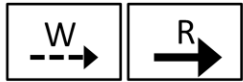
... well



## Self-stabilization

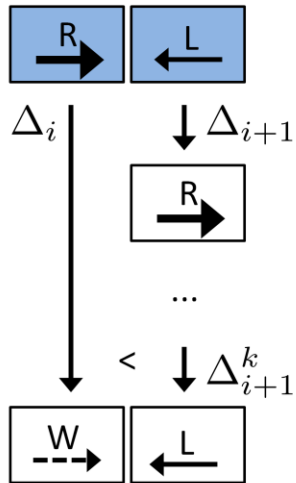


$$\Delta_i \downarrow = \downarrow \Delta_{i+1}$$



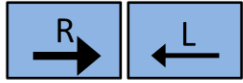
requires simultaneity: two sided constraint!

# Self-stabilization



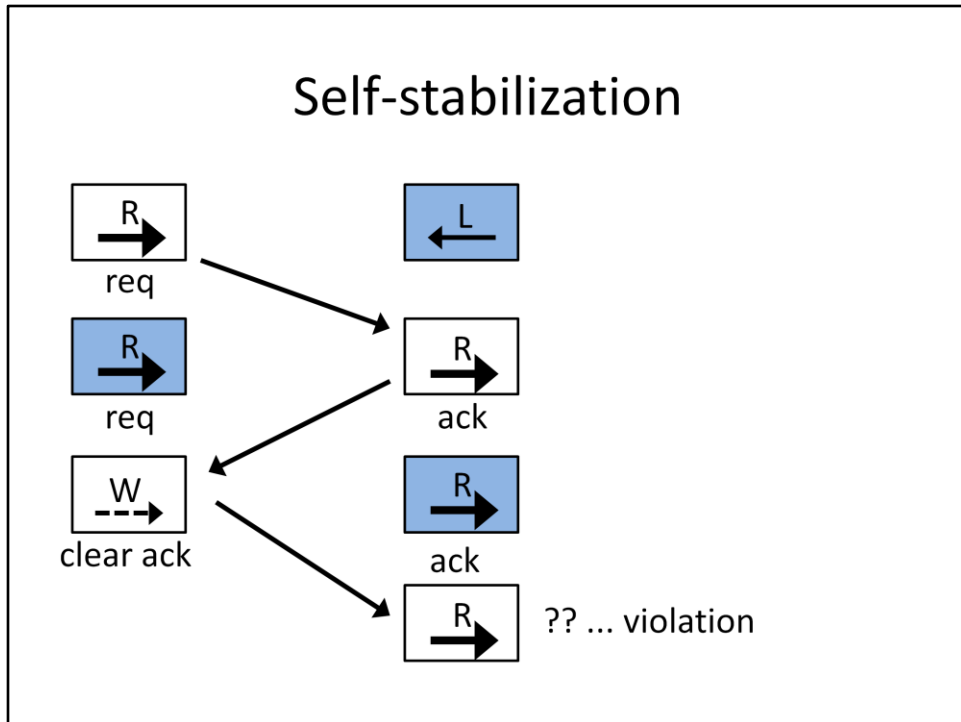
... one sided

# Self-stabilization



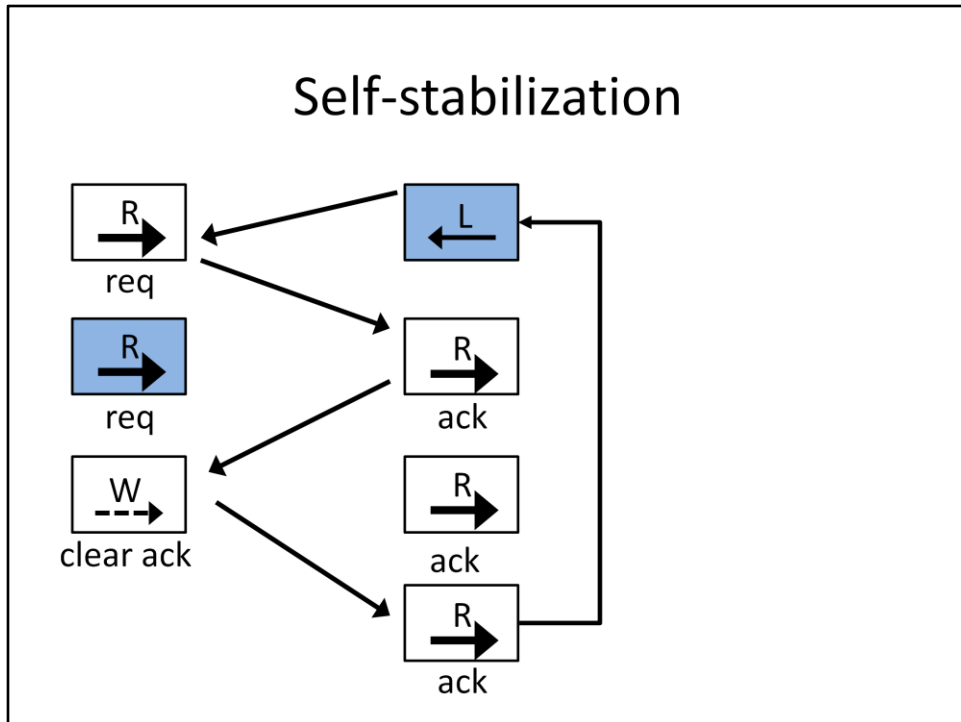
... without timing?

# Self-stabilization



violation since right node with state „R ack“ makes a step to „R“ in response to left one, not looking to left one. This could result in unstable change.

## Self-stabilization



no violation workaround: right node with „R ack“ waits with removing ack until it looks left again.

Beyond classical circuit design  
lecture 3.5

Circuit model

## Further Reading

Alain J. Martin: *Synthesis of Asynchronous VLSI Circuits*. Tech report California Institute of Technology, 1991.

Alain J. Martin and Mika Nyström: *Asynchronous techniques for system-on-chip design*. Proceedings of the IEEE Volume 94, Issue 6:1089 - 1120, June 2006.

## Binary, event based model

here [Alain Martin]:

low-level: production rules.

high-level: communicating hardware processes.



Low-level Specifications

Production rules

## Production rules

variable/port: from a finite alphabet  $V$

transition: variable + up/down

production rule: Boolean guard  $\rightarrow$  transition

$$x \wedge y \rightarrow z \uparrow$$

$$\neg(x \wedge y) \rightarrow z \downarrow$$

## Production rules

$$x \wedge y \rightarrow z \uparrow$$

$$\neg(x \wedge y) \rightarrow z \downarrow$$

typically rule-pairs

**non-interference:** per rule-pair  $\neg(Bu \wedge Bd)$

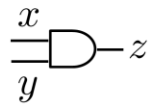
**no self-reference:** per rule

# Gate

gate = rule-pair

**combinational** (NOT, 2AND, 2OR, AOIs, ...)

$$Bu \leftrightarrow \neg Bd$$



$$x \wedge y \rightarrow z \uparrow$$

$$\neg(x \wedge y) \rightarrow z \downarrow$$

AOI = AND-OR-INVERT gate

# Gate

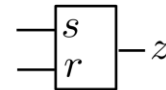
gate = rule-pair

## state holding

set-reset latch

$$s \rightarrow z \uparrow$$

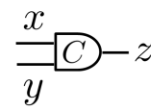
$$r \rightarrow z \downarrow$$



2C-Element

$$x \wedge y \rightarrow z \uparrow$$

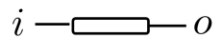
$$\neg x \wedge \neg y \rightarrow z \downarrow$$



set-reset latch: unspecified what happens if  $s=r=1$ . Is disallowed by non-interference. When using this gate: Make sure that non-interference is valid in all executions.

# Wire

= special gate



$$i \rightarrow o \uparrow$$

$$\neg i \rightarrow o \downarrow$$

## Production rules

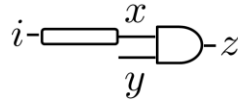
circuit = algorithm = set of production rules

$$Au : x \wedge y \rightarrow z \uparrow$$

$$Ad : \neg(x \wedge y) \rightarrow z \downarrow$$

$$Du : i \rightarrow x \uparrow$$

$$Dd : \neg i \rightarrow x \downarrow$$



environment = set of production rules

## Execution

global state  $s : V \mapsto \{0, 1\}$

enabled rule, step

execution  $(s_n)_{n \geq 0}$

constraints: (weak) fairness, partial order, timed



## Hardware design

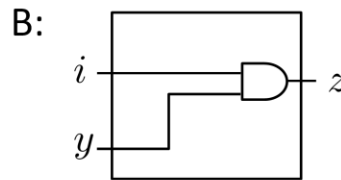
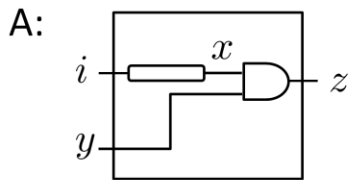
Given basic building blocks, implement the specification.

## Circuit A implements circuit B

observable variables  $\mathcal{O}$

trace inclusion

$$E_A \upharpoonright \mathcal{O} \subseteq E_B \upharpoonright \mathcal{O}$$



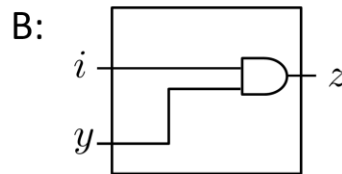
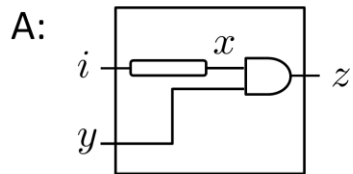
if circuit A produces behavior that could be from circuit B as well, A implements B. Typically B is a circuit specification. The executions of B are the allowed executions. If A's executions all are in the set of the allowed executions we say A implements (specification) B.

## Circuit A implements circuit B

$i \uparrow x \uparrow i \downarrow y \uparrow z \uparrow \mapsto$

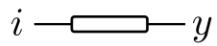
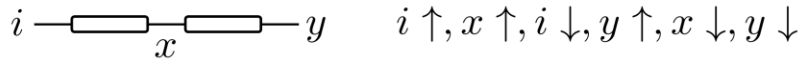
$i \uparrow i \downarrow y \uparrow z \uparrow$

-> A does not implement B



# Mind...

wire + wire "is" not a (long) wire

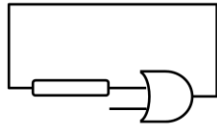


„is“ here means: implements in both directions: A is B if A implements B and B implements A.

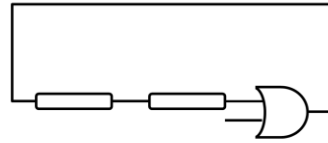
# Mind...

wire + wire "is" not a (long) wire

->



vs.

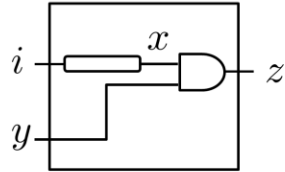


oscillations?! [hw]

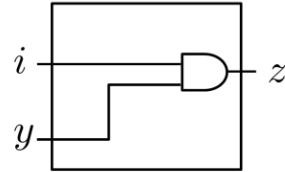
“implements”

Simulation.

A:



B:

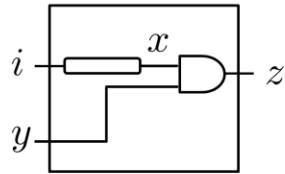


$i \uparrow$

“implements”

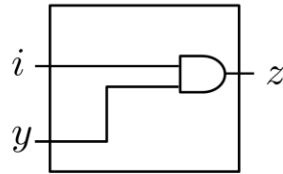
Simulation.

A:



$i \uparrow$

B:

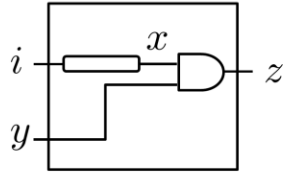


$i \uparrow$

# “implements”

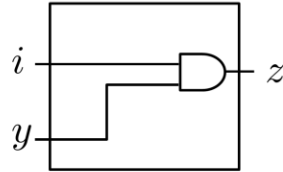
Simulation.

A:



$i \uparrow$

B:



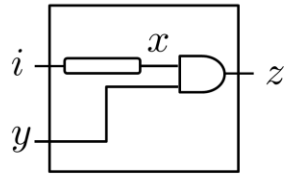
$i \uparrow i \downarrow$



“implements”

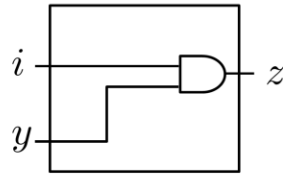
Simulation.

A:



$i \uparrow i \downarrow$

B:

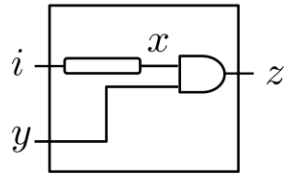


$i \uparrow i \downarrow$

“implements”

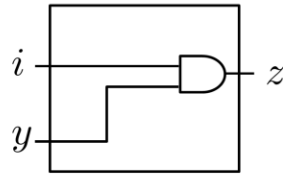
Simulation.

A:



$i \uparrow i \downarrow$

B:

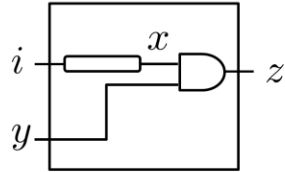


$i \uparrow i \downarrow y \uparrow$

# “implements”

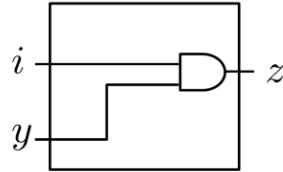
Simulation.

A:



$i \uparrow i \downarrow y \uparrow$

B:

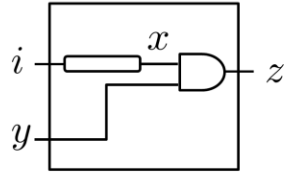


$i \uparrow i \downarrow y \uparrow$

# “implements”

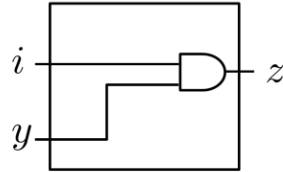
Simulation.

A:



$i \uparrow i \downarrow y \uparrow$

B:

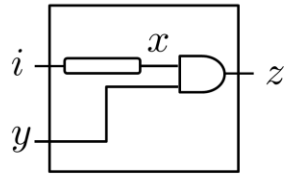


$i \uparrow i \downarrow y \uparrow i \uparrow$

# “implements”

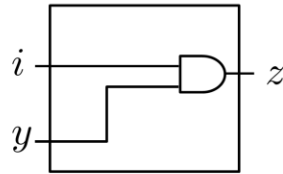
Simulation.

A:



$i \uparrow i \downarrow y \uparrow i \uparrow$

B:

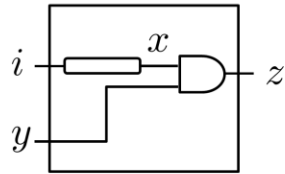


$i \uparrow i \downarrow y \uparrow i \uparrow$

# “implements”

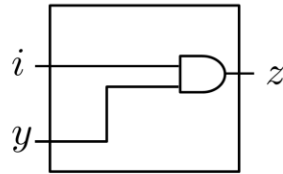
Simulation.

A:



$i \uparrow i \downarrow y \uparrow i \uparrow$

B:

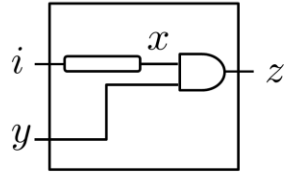


$i \uparrow i \downarrow y \uparrow i \uparrow z \uparrow$

# “implements”

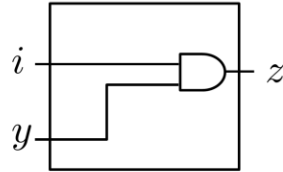
Simulation.

A:



$i \uparrow i \downarrow y \uparrow i \uparrow x \uparrow z \uparrow$

B:



$i \uparrow i \downarrow y \uparrow i \uparrow z \uparrow$

## “implements”

A can simulate B.

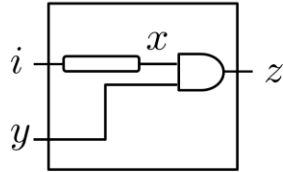
Game rules:

- B makes a sequence of steps:  
non-observables with ending observable
- A makes a sequence of steps:  
non-observables with same ending  
observable

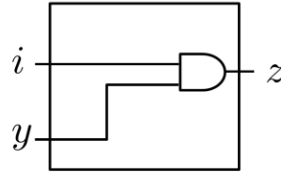


# “implements”

A can simulate B  $\rightarrow$  B implements A [hw]



$i \uparrow i \downarrow y \uparrow i \uparrow x \uparrow z \uparrow$



$i \uparrow i \downarrow y \uparrow i \uparrow z \uparrow$

## “implements”

A can simulate B  $\rightarrow$  B implements A [hw]

Simulation is an efficient test for  
implementation.

## “implements”

A can simulate B  $\rightarrow$  B implements A [hw]

Simulation is an efficient test for  
implementation.

Is “can simulate” also necessary?

“implements”

A can simulate B  $\leftarrow$  B implements A ?

## “implements”

$$\mathcal{O} = \{a, b, c, d\}$$

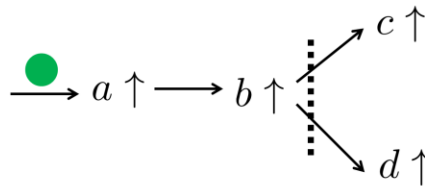
$$\top \rightarrow a \uparrow$$

$$[\perp \rightarrow a \downarrow]$$

$$a \rightarrow b \uparrow$$

$$b \wedge \neg d \rightarrow c \uparrow$$

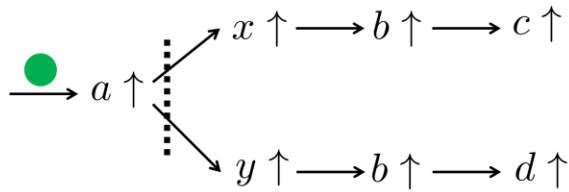
$$b \wedge \neg c \rightarrow d \uparrow$$



left side: circuit. right side: Petri-net-like graphical representation of the causal structure (what event causes what event) of the circuit. the green dot represents the initial event triggered. the dotted bar means that not both branches can be taken, but only one of them (mutually exclusive)

“implements”

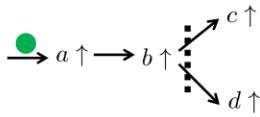
$$\mathcal{O} = \{a, b, c, d\}$$



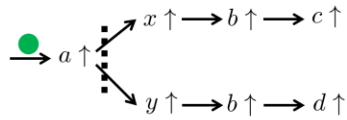
Note that again we could write the circuit for this graphical representation in our circuit model notation.

## “implements”

Circuit A



Circuit B

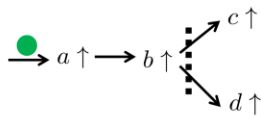


B implements A and A implements B.  
A can simulate B

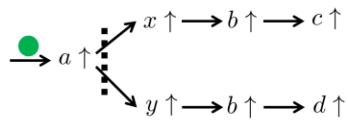
A implements B and B implements A: we know this since the only 2 executions projected to observable variables in both circuits are: a goes high, b goes high, c goes high and a goes high, b goes high, d goes high.

## “implements”

Circuit A



Circuit B



B implements A      and      A implements B.  
A can simulate B      but      B **cannot** simulate A.

The problem is that circuit A may decide later than B which branch it takes.



“implements”

-> other notions of “can simulate”

There are more powerful notions where player A can reverse some of its steps (e.g., it can reverse the last  $k$  steps). Still such simulation relations are not equivalent with the implementation relation.