Computational Geometry and Geometric Computing
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Exercise 3

Motivation

We practise our knowledge of floating point arithmetic.

Doubles

What is the largest number representable as a double, the smallest positive number, the smallest normalized positive number?

Some Computations are Exact

Let $a, b \in F$ with $\frac{1}{2} \leq \frac{a}{b} \leq 2$. Show that $a \ominus b = a - b$. This was first observed by Sterbenz.

Doubles and Orientation

Assume for this exercise that point coordinates are doubles in $[1/2, 1]$. Show

- $orient(p, q, r) = 0$ implies $floatorient(p, q, r) = 0$.
- $floatorient(p, q, r) \neq 0$ implies $orient(p, q, r) = floatorient(p, q, r)$.
- What does this mean for the geometry of float-orient?
- Can you find examples that make the floating point implementation of the convex hull algorithm crash when point coordinates are restricted to doubles in $[1/2, 1]$?

Error Analysis

Assume that a point p is given by its homogeneous coordinates (px, py, pw). Assuming $sign(aw \cdot bw \cdot cw) = 1$, we have

$$orient(a, b, c) = sign(aw \cdot (bx \cdot cy - by \cdot cx) - bw \cdot (ax \cdot cy - ay \cdot cx) + cw \cdot (ax \cdot by - ay \cdot bx)).$$

Compute the d-value and m-value of this expression.

A High Precision Computation of π

Show how to compute π with an error less than 2^{-200}.

Linear Kernel

In class we discussed the concept of a linear kernel and several models of it. The notes contain a `C++` implementation. Give an implementation in a programming language of your choice (preferably not `C++`).

Rational Points on a Circle

In class we showed that for any rational point $p = (p_x, p_y)$ on the unit circle there is a rational a such that

$$(p_x, p_y) = \left(\frac{2a}{a^2+1}, \frac{a^2-1}{a^2+1} \right).$$

In order to find a rational point in direction α, we therefore need to find a rational a such that

$$a \approx \frac{1}{\cos\alpha} + \sqrt{\frac{1}{\cos^2\alpha} - 1}.$$

Show how to find such an approximation with error less than 2^{-t} by binary search.

Have fun with the solution!