Computational Geometry and Geometric Computing
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Exercise 8

Motivation

We consider planar curves and their intersection. Furthermore, we introduce a new method for root isolation of univariate polynomials.

Intersections of Planar Curves

- Consider the circles

$$\begin{aligned} C_1 &= V((x-1)^2 + y^2 - 2) \\ C_2 &= V(x^2 + (y+2)^2 - 3) \\ C_3 &= V(x^2 + y^2 - 4.1599) \end{aligned}$$

  How many different pairwise intersection points exist? Is $C_1 \cap C_2 \cap C_3 = \emptyset$? (proof!)

- For which values of λ is

$$C_1 \cap C_2 \cap C^{(\lambda)} \neq \emptyset,$$

  where $C^{(\lambda)} = V(x^2 + y^2 - \lambda)$?

- Determine all real valued intersection points of the curves $C := V(x^3 + x^2y - 2x^2 - 3xy - 3y^2 + 6y)$ und $D := V(x^4 - 2x^2y - 2x^2 - 3y^2 + 6y)$!

- For this exercise, we recommend to use the web demo at `http://exacus.mpi-sb.mpg.de/cgi-bin/xalci.cgi` (You do not have to prove your observation!). Consider the parameterized curve

$$C^{(\lambda_1,\ldots,\lambda_4)} := V(y^4 - y^3 + 2x^2y^2 + 3x^2y + x^4 + \lambda_1 x^2 y + \lambda_2 xy + \lambda_3 x + \lambda_4)$$

  1. Start with $\lambda_1 = \ldots = \lambda_4 = 0$. Where does $C^{(0,\ldots,0)}$ have a singular point (self-intersection)?
  2. Set $\lambda_1 = 0.001$, is there a significant change ?
  3. Set $\lambda_2 = 0.001$ as well. What is the number of singular points ? Is there a significant change at all ?
  4. What happens if we set $\lambda_3 = 0.001$ as well ?
  5. Set $\lambda_4 = 0.001$. Is there a self intersection ?
  6. Choose some arbitrary values for $\lambda_1, \ldots, \lambda_4$. What do you observe? Does $C^{(\lambda_1,\ldots,\lambda_4)}$ have any self intersections?

Sturm Sequences

We consider the polynomial $f(x) = 8 - 4x + 6x^2 - 3x^3 - 2x^4 + x^5$.

- Compute a Sturm sequence $\mathcal{S} = \{S_0, ..., S_k\}$ for f and evaluate $\mathcal{S}$ at $x = -3, -2, -1, 0, 1, 2, 3$.
- What can you say about the number of real roots within $(-3, 3)$?
- Formulate a general algorithm based on Sturm's theorem to isolate the real roots of an arbitrary polynomial g with rational coefficients (Take care about the prerequisites in Sturm's theorem!).
- Isolate the roots of f.

Have fun with the solution!