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Computational Geometry and Geometric Computing
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Exercise 9

Motivation

Descartes' Rule of Sign, Root Isolation and Möbius transformations.

Descartes' Rule of Sign

- Let $f = \sum_{i=0}^{n} a_i x^i$ be a polynomial with n real roots and $f(0) \neq 0$. Show that $a_i = 0$ implies that $a_{i-1} \cdot a_{i+1} < 0$.
- Determine isolating intervals for the real roots of

  $$f = 30x^5 + 95x^4 - 2x^3 - 53x^2 - 4x + 6$$

  by the use of the VCA algorithm.

Möbius Transformations

Show that, for $\lambda \in \mathbb{R}$ and $\mu \in \mathbb{R}\backslash\{0\}$, each of the following transformations

$$t_\lambda : z \to z + \lambda$$

$$h_\mu : z \to \mu z$$

$$r : z \to \frac{1}{z}$$

- is bijective on $\bar{\mathbb{C}} := \mathbb{C} \cup \infty$.
- maps general circles to general circles. We call a subset $C \in \bar{\mathbb{C}}$ in the complex space a general circle if C is either a circle or a line with ∞.
- preserves angles, that is, given two curves C_1 and C_2 in $\mathbb{C}$ that intersect in a point ξ with an included angle α and let T be one of the transformations from above then the curves $T(C_1)$ and $T(C_2)$ intersect in $T(\xi)$ with an included angle α as well.

  Hint: Consider two lines $L_i = \overline{a_i b_i}$, $i = 1, 2$, passing two distinct points $a_i, b_i \in C_i$. Compare the angle between the lines $\tilde{L}_i := \overline{T(a_i), T(b_i)}$ and that included by L_1 and L_2! What happens if a_i and b_i converge to ξ?

Have fun with the solution!