Computational Geometry and Geometric Computing
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Exercise 10

Motivation

We continue our study of planar curves and root isolation methods.

Homogeneous Bivariate Polynomials

Call a bivariate polynomial $f(x, y)$ homogeneous, if the degree of all terms in f is the same. So $x^2 - y^2 - xy$ and $xy^3 - x^2y^2 + y^4$ are homogeneous, but $x^2 + y$ is not.

- Use `http://exacus.mpi-sb.mpg.de/cgi-bin/xalci.cgi` to plot the curves $x^2 - y^2 - xy = 0$ and $xy^3 - x^2y^2 + y^4 = 0$, and $x^2 - y^2 = 0$. Formulate a conjecture about the shape of vanishing sets of homogeneous polynomials.
- Show that any homogeneous polynomial factors into linear factors of the form $ax + by$ with $a, b \in C$.

The Shape of a Curve near the Origin

Let f be a bivariate polynomial and let $f^{*}*$ be the homogeneous polynomial formed by the lowest order terms of f. For $f(x, y) = y^3 + x^2 - y^2 + 2xy$, f^* consists of all terms of degree 2, i.e. $f^* = x^2 - y^2 + 2xy$.

- Experiment with different f's. Use `http://exacus.mpi-sb.mpg.de/cgi-bin/xalci.cgi` to plot the curves $f(x, y) = 0$ and $f^*(x, y) = 0$ near the origin. Formulate a conjecture.
- Prove the conjecture.

0.1 Early Termination of Descartes Algorithm

Consider $p(x) = x^2 + \delta^2 = (x - \mathbf{i}\delta)(x + \mathbf{i}\delta)$ with $\delta \approx 0$. This polynomial has a pair of conjugate complex roots at $\pm\mathbf{i}\delta$ and hence separation 2δ. However, $var(p, (-1, 1)) = 2$ and $var(p, (-1, 0)) = var(p, (0, 1)) = 0$. Verify these statements. Thus the algorithm ends with intervals of length 1/2, although the separation may be arbitrarily small.

Have fun with the solution!