Computational Geometry and Geometric Computing
Eric Berberich,
Kurt Mehlhorn,
Michael Sagraloff

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Exercise 11

Bitstream Descartes for multiple roots

We "extend" the Bitstream Descartes to polynomials with multiple roots: Let f be a polynomial with real coefficients and k denote the maximal multiplicity of a root of f.

- Show that there exists a $w_0 > 0$ such that for all intervals of size $w(I) < w_0$ it holds that $\mathrm{var}(f, I) \leq k$. Give a bound on w_0 in terms of the separation of f (= minimal distance between two distinct roots).
- We call a polynomial *generic* iff it has a root of multiplicity $k + 1 = \deg \gcd(f, f')+1$. Show that, for $k > 0$, each generic polynomial has exactly one multiple root z. Furthermore, show that z is real and all remaining roots of f are simple.
- Now, f is a polynomial with bitstream coefficients. Furthermore, we assume that $k := \deg \gcd(f, f')$ is known and that we can ask for an arbitrary good approximation $\tilde{f}^*$ of the square-free part $f^* := f/\gcd(f, f')$ of f. Formulate an algorithm to
  1. determine isolating intervals $I_1, \ldots, I_m$ for the real roots of f.
  2. refine the intervals I_j to any specified size.
  3. determine whether f is generic or not.
  4. determine which of the intervals I_j contains the unique multiple root of f.

Topology of a Planar Curve

Determine the topology of the planar curve $C := V_{\mathbb{R}}(x^3 - 2xy + 2y^2 + x^2)$, that is, compute an isocomplex for C.

Why is this argumentation wrong?

Let C be a planar algebraic curve. We are interested in a shearing of C such that the transformed curve C' is in general position, that is, no two critical points are co-vertical. We want to show that all but finitely many shearing directions induce a curve C' in general position:

It suffices to find a direction $\phi \in [0, 2\pi)$ such that each line pointing toward the direction ϕ does not pass two or more critical points of C. There exists only finitely many critical points $p_1, \ldots, p_m$ of C. Let ϕ_i, $i = 1, \ldots, \binom{m}{2}$ denote the directions defined by each pair of *critical points, then, each direction* $\phi \neq \phi_i$ *defines a shearing which induces a curve in general position. This shows our claim.*

Have fun with the solution!