Computational Geometry and Geometric Computing
Eric Berberich,
Kurt Mehlhorn,
Michael Sagraloff

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Discussion on February 3rd.

Exercise 12

Isocomplexes

Can you give an upper bound $B(n)$ in terms of the degree $n = \deg f$ such that each algebraic curve $C := V_{\mathbb{R}}(f)$ of degree n can be linearly approximated by a (stable) isocomplex of size $B(n)$, that is, the number of vertices and edges is smaller than $B(n)$. Give a proof idea. Can you also give lower bounds, i.e., by finding examples (family of curves) where you always need a certain number of simplices.

Topology and roots

Let (x_0, y_0) be a point on an algebraic curve $C = V_{\mathbb{R}}(f)$. Show that if (x_0, y_0) is incident to $2k$ branches of the curve, the resultant $\mathrm{res}(f, f_y, y)$ has a root of multiplicity at least $k-1$ at $x = x_0$.

Sweeping algebraic curves

Show how to implement the operations needed for the sweep line algorithm for algebraic curves assuming that you are able to analyze single algebraic curves and pairs of them (as shown in the lecture for curves in generic position). Hint 1: The required operations are Make_x_monotone, Compare_x, Compare_xy, Point_position, Order_to_right, and Intersection (you may assume that any two curves intersect in finitely many points). Hint 2: Let Make_x_monotone "nicely" decompose the input curves.

Have fun with the solution!