Universität des Saarlandes

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Models of Computation, an Algorithmic Perspective

Assignment 13 Fri 28.1.2011

This assignment is **due on February 2/4** in your respective tutorial groups. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Exercise 1 [Shortest Path:] We covered an external memory algorithms for BFS in undirected graphs in class. BFS is tantamount to shortest paths where all edges have length 1. We now allow edge lengths in $\{1, d\}$ where d is a small constant. Generalize the algorithm and its analysis.

Selection: We are given a S and an integer i with $1 \leq i \leq |S|$. The goal is to find the i-th largest element in S. S is ordered by $\leq$.

Exercise 2 Recapitulate the sequential solution. We define a recursive procedure $select(S, i)$.

- randomized solution: if $|S| = 1$ (and hence $i = 1$) return the unique element in S. If $|S| > 1$, choose a random element in S, call it x, and split S into $S_< = \{s \in S; s < x\}$, $\{x\}$, and $S_> = \{s \in S; s > x\}$. If $i \leq |S_<|$, return $select(S_<, i)$, if $i = |S_<| + 1$, return x, and otherwise return $select(S_>, i - |S_<| + 1)$. Randomized select works in linear time.
- deterministic solution: divide S into groups of size 5 and determine the median of each group. Let $m_1, \ldots, m_k$ be the medians. Let m^* be the median of the medians; use the procedure recursively to find m^*. Then proceed as in the deterministic solution with $x = m^*$. Deterministic select works in linear time.

Generalize to external memory, PRAM, and multi-core.