max planck institut informatik

SAARLAND UNIVERSITY

COMPUTER SCIENCE

Prof. Dr. Benjamin Doerr **Winter 2012/13**

Exercises for Randomized Methods in Computer Science

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws12/rmcs/

Assignment 8 Due: Wednesday, December 19, 2012

Exercise 1 Please read the paper

B. Doerr, E. Happ, C. Klein.
Crossover can provably be useful in evolutionary computation.
Theoretical Computer Science 425 (2012), 17-33.

You may omit the lower bound for the optimization of the mutation-only EA and the analyses of the second and third crossover operator.

Denote by $\ell(P)$ the largest integer $\ell \in [0..n-1]$ such that P contains, for each pair of vertices that can be connected by a shortest path having at most ℓ edges, such a path. Here and in the following, by a "shortest path having at most ℓ edges" we mean a path between two vertices (i) such that there is no shorter path between these two vertices and (ii) that consists of at most ℓ edges.

Answer the following questions.

a) In the evolutionary algorithms regarded, the individuals are not represented by bit-strings. What is the analogue of the RLS variation operator "flip a random bit" in this setting?

   How does the mutation operator used in the paper imitate the property of the (1+1) EA with bit-string representation that more than one bit can be flipped?

   Give a simple argument why both EA and the GA regarded in the paper converge to an optimal solution.

b) Imagine that the EA given in the paper (that is, the algorithm not using crossover) starts with a population P such that $\ell(P) = \ell$, but which contains no paths containing more than ℓ edges.

   How long does it take until it found all shortest paths of $\ell + 1$ edges? How do you explain the discrepancy of n times this number and the optimization time of the EA?

c) Let us now analyze the GA, that is, the algorithm using crossover. Which two facts make crossover increasingly efficient during the run of the GA? What is a weak point of crossover (compared to mutation)? What would be the run-time of the GA if only crossover is used and no mutation?

d) Describe how $\ell(P)$ developes over time in a typical run of the GA.