

Exercise 12: ~~Weed~~ Weak models

Task 1: Hyper, hyper!

Recall that a hypergraph is a graph where edges may comprise more than 2 nodes. The degree of a hyperedge is the number of nodes in it. Consider a hypergraph of maximum degree d .

- a) Generalize maximal matchings and vertex covers to hypergraphs.
- b) Give a d -approximation to vertex cover on hypergraphs of maximum edge degree d and prove its approximation ratio. (Hint: Use a maximal matching!)
- c) Show how to compute a maximal matching on hypergraphs of maximum node and edge degrees Δ and d , respectively, within $\mathcal{O}((d\Delta)^d)$ rounds. Here, we assume that hyperedges are represented as nodes as well, i.e., a hyperedge is a node connected to all the nodes in the hyperedge; nodes know whether they represent a node or a hyperedge. (Hint: Let hyperedges propose to their constituent nodes one by one, in ascending order. Any hyperedge whose node does not accept is out for this iteration. Show that whenever a hyperedge is kicked out, an adjacent one makes it to the next stage. Conclude that some node that is not too far away gets covered and bound the number of such nodes.)

Task 2: Colors, all these beautiful colors!

- a) Show that an edge coloring in the port numbering model is insufficient to obtain a node coloring!
- b) Suppose we have an anonymous graph without port numbers, but with a node coloring. Communication is synchronous, by sending a message to each color and receiving a multiset of messages (i.e., the messages by the neighbors of this color, counting multiplicity) from each color. Show that this is insufficient to obtain an edge coloring!
- c) Show how to construct an edge coloring from a node coloring in the port numbering model!

Task 3*: ... huh?

- a) Meditate on the nature of the course and the insights you gained.
- b) Extract questions that will guide the true seeker of knowledge to these insights.
- c) Pose your questions in the exercise group for the others to contemplate.¹

¹More profanely said, figure out what was important in the course, come up with viable exam questions, and let us know. If they are good, we may ask them, making everyone's life better and happier.