Exercise 2: Flirting with Synchrony and Asynchrony

Task 1: Growing Balls

Denote by $B(v, r)$ the ball of radius r around v, i.e., $B(v, r) = \{u \in V : \mathrm{dist}(u, v) \leq r\}$. Consider the following partitioning algorithm.

Algorithm 1 Cluster construction. $\rho \geq 2$ is a given parameter.

while there are unprocessed nodes **do**
    select an arbitrary unprocessed node v;
    $r := 0$;
    while $|B(v, r + 1)| > \rho|B(v, r)|$ **do**
        $r := r + 1$
    end while
    makeCluster($B(v, r)$) // all nodes in $B(v, r)$ are now processed
    remove all cluster nodes from the current graph
end while
select intercluster edges

a) Show that Algorithm 1 constructs clusters of radius at most $\log_\rho n$!

b) Show that Algorithm 1 will produce at most ρn intercluster edges!

c) For $k \in \{1, \ldots, \lceil \log n \rceil\}$, determine an appropriate choice $\rho(k)$ and use it to prove Corollary 2.14!

Task 2: Showing Dijkstra, Bellman & Ford the Ropes

a) Show that if the asynchronous Bellman-Ford algorithm from the lecture is executed synchronously, it sends only $\mathcal{O}(|E|D)$ messages.

b) Use this to construct an asynchronous BFS tree construction algorithm of time complexity $\mathcal{O}(D)$ that uses $\mathcal{O}(|E|D)$ messages and terminates. You may assume that D is known here.

c) Reduce the message complexity of the synchronous algorithm from b) to $\mathcal{O}(|E|)$ by eliminating "useless" messages. Do the same for the asynchronous algorithm!

d) Modify the synchronous algorithm such that the root will know that the construction is complete at most $\mathcal{O}(D)$ rounds after this is the case. Use only $\mathcal{O}(nD)$ additional messages! Do the same for the asynchronous algorithm! (Hint: make it such that if the root *doesn't* hear anything for 2 consecutive rounds, the construction is finished.)

e) Use this to construct an asynchronous BFS tree construction algorithm of time complexity $\mathcal{O}(D)$ that uses $\mathcal{O}(|E| + nD)$ messages and terminates.

Task 3*: Liaison with Leslie Lamport

a) Look up what Lamport causality, Lamport clocks, and Lamport vector clocks are!

b) Contemplate their relation to synchronizers and what you've learned in the lecture!

c) Discuss your findings in the exercise session!