Advanced Courses CS, Summer Term 2009

# Music Processing

Meinard Müller

Max-Planck-Institut für Informatik
Campus E1 4, 66123 Saarbrcken, Germany
meinard@mpi-inf.mpg.de

## Dynamic Time Warping (DTW)

**Problem 1**

Let $\mathcal{F} = \{\alpha, \beta, \gamma\}$ be a feature space. Furthermore, let $c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$ be local cost measure defined by

$$c(x,y) := 1 - \delta_{xy} = \begin{cases} 0 & \text{falls } x = y, \\ 1 & \text{falls } x \neq y, \end{cases}$$

for $x, y \in \mathcal{F}$. Specify the DTW distances $\mathrm{DTW}(X,Y)$, $\mathrm{DTW}(X,Z)$ und $\mathrm{DTW}(Y,Z)$ for the following sequences:

$$X = \alpha\beta\gamma\alpha\beta\gamma, \quad Y = \gamma\alpha\beta, \quad Z = \alpha\alpha\gamma\alpha.$$

**Problem 2**

Let $F = \mathbb{R}$ be a feature space and $c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$ the local cost measure defined by $c(x,y) = |x-y|$, $x, y \in \mathbb{R}$. Furthermore, let $X = (1,7,4,4,6)$ and $Y = (1,2,2,7)$. Execute the algorithm by hand as described in the lecture for computing $\mathrm{DTW}(X,Y)$. In particular, specify the cost matrix $C$ and the accumulated cost matrix $D$. Furthermore, specify an optimal warping path. Is there another optimal warping path?

**Problem 3**

Let $\mathcal{F}$ be a feature space and $c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$ be a symmetric local cost measure, i. e., $c(x,y) = c(y,x)$ for all $x, y \in \mathcal{F}$. Show that in this case the DTW distance is symmetric as well. Furthermore, show that the DTW distance generally does not satisfy the triangle inequality.

**Problem 4**

In this problem, the multiscale approach for DTW (MsDTW) is to be analyzed in more detail. Let $X = (x_1, x_2, \ldots, x_N)$ and $Y = (y_1, y_2, \ldots, y_M)$ be sequences of length $N$ and $M$. For simplicity, we assume that $N = M = 2^n$ for a natural number $n \in \mathbb{N}$. Let $A^{\mathrm{DTW}}(N) = N^2$ denote the number of evaluations of the local cost measure that are required in the classical DTW algorithm.

In the following, we assume that the MsDTW-approach is performed recursively with $f_1 = f_2 = \ldots = f_n = 2$. Let $A^{\mathrm{MsDTW}}(N)$ denote the number of evaluations of the local cost measure that are required in the MsDTW algorithm. Specify a possibly small upper bound for $A^{\mathrm{MsDTW}}(N)$. What can be said about the memory requirements?

(Note: In the analysis, the operations required for computing the coarsened sequences $X_1, X_2, \ldots, X_n$ und $Y_1, Y_2, \ldots, Y_n$ are left unconsidered. )