Geometric Modeling

Summer Semester 2010

Introduction

Motivation · Topics · Basic Modeling Techniques

UNIVERSITÄT DES SAARLANDES

CLUSTER OF EXCELLENCE

max planck institut informatik

Today...

Topics:

- Formalities & Organization
- Introduction: Geometric Modeling
- Mathematical Tools (1)

Today...

Topics:

- Formalities & Organization
- Introduction: Geometric Modeling
  - Motivation
  - Overview: Topics
  - Basic modeling techniques
- Mathematical Tools (1)

Motivation

Motivation

This lecture covers two related areas:

- Classic geometric modeling
- Geometry processing

Common techniques (math, models, terminology), but different goals

Geometric Modeling

Geometric Modeling:

- You start with a blank screen, design a geometric model
- Typical techniques:
  - Triangle meshes
  - Constructive Solid Geometry (CSG)
  - Spline curves & surfaces
  - Subdivision surfaces
- Goal is *interactive modeling*
- Mathematical tools are designed *with the user in mind*

Geometry Processing

Geometry Processing

- You already have a geometric model
  - Typically from a 3D range scanner (read: not nice)
  - You need to process & edit the geometry
  - The original model has not been build with the user in mind (stupid range scanner)
- Typical techniques:
  - Noise removal, filtering
  - Surface reconstruction
  - Registration
  - Freeform deformation modeling
  - Statistical analysis (features, symmetry, hole-filling etc...)

Our Perspective

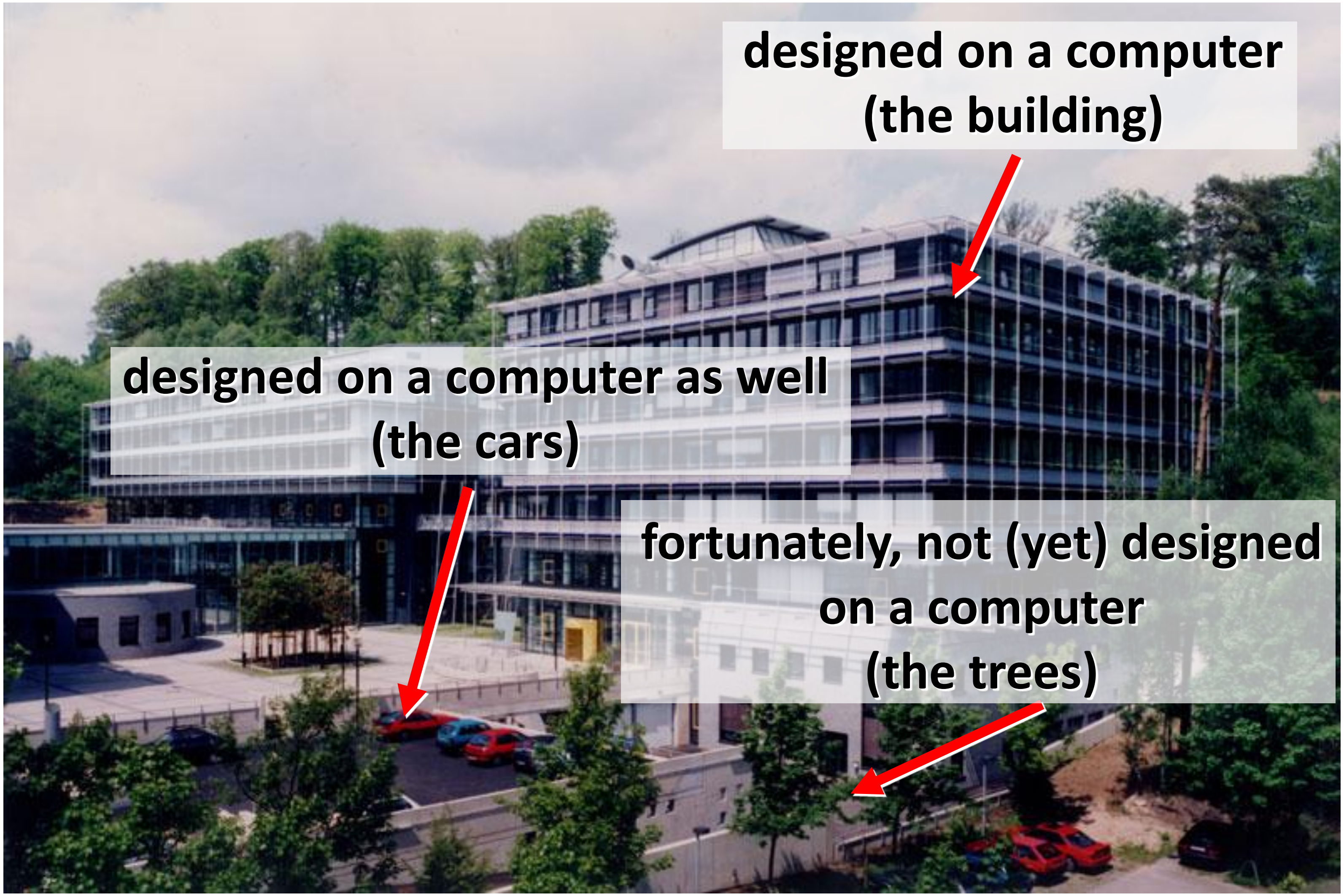
The perspective of this lecture:

- The basic mathematical tools for handling geometry are the same
- Different usage, adaptation, specific algorithms
- We will discuss
  - The basic concepts and tools
(mathematical foundation, representations, basic algorithms)
  - ...and applications in both areas.

Examples:

Geometric Modeling

The Modern World...



(c.f. Danny Hillis, Siggraph 2001 keynote)

Impact of Geometric Modeling

We live in a world designed using CAD

- Almost any man-made structure is nowadays planed and designed using computers
  - Architecture
  - Commodities: Chairs, furniture, your microwave & toaster
  - Your car (in case you have one, but probably the bike as well) – spline curves have actually been invented in the automotive industry
  - Typesetting
- <advertising> Our abilities in geometric modeling shapes the world we live in each day. </advertising>

Different Modeling Tasks

CAD / CAM

- Precision Guarantees
- Handle geometric constraints exactly (e.g. exact circles)
- Modeling guided by rules and constraints

Different Modeling Tasks

Photorealistic Rendering

- Has to “look” good
- Ad-hoc techniques are ok
- Using textures & shaders to “fake” details
- More complexity, but less rigorous

Just two examples, lots of stuff in between...

Examples:

Geometry Processing

Geometry Processing

A rather new area

- Motivation: 3D scanning
  - You (your company) can buy devices that scan real world 3D objects
  - You get (typically) clouds of measurement points
- Many other sources of geometry as well:
  - Science (CT, [F]MRI, ET, Cryo-EM, ...)
  - 3D movie making
  - The design department of your company has dozens of TByte of “polygon soup”...
  - Crawl the internet
- Need to process the geometry further

Photoshopping Geometry

Geometry Processing:

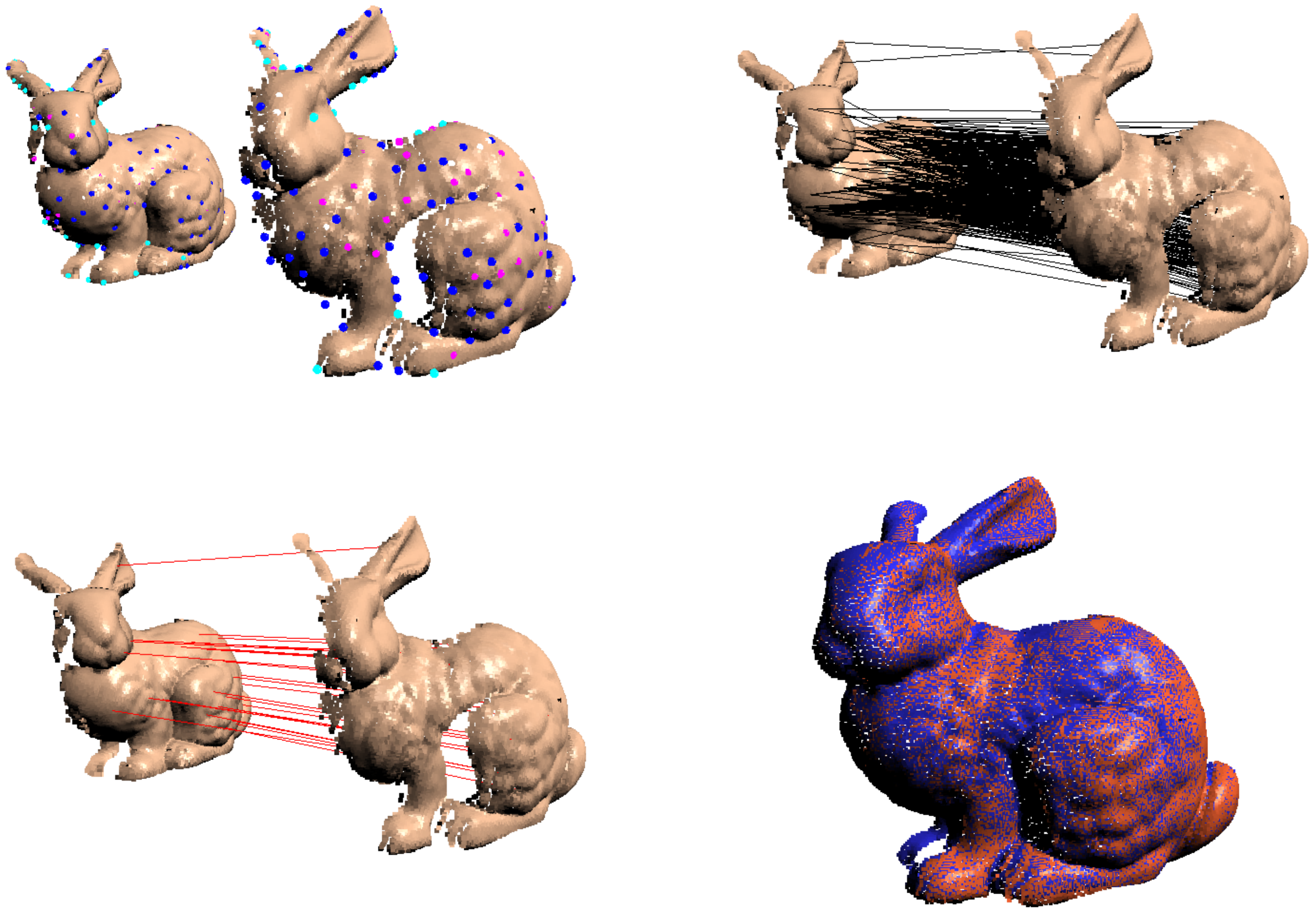
- Cleanup:
  - Remove inconsistencies
  - Make watertight (well defined inside/outside, for 3D printers)
  - Simplify – keep only the main “structure”
  - Remove noise, small holes, etc...
- Touch-up /Edit:
  - Texturing, painting, carving
  - Deformation
  - Stitch together pieces
- Lots of other stuff – similar to image processing

Example

Example: The Stanford “Digital Michelangelo Project”

[Levoy et al.: The Digital Michelangelo Project, Siggraph 2000]

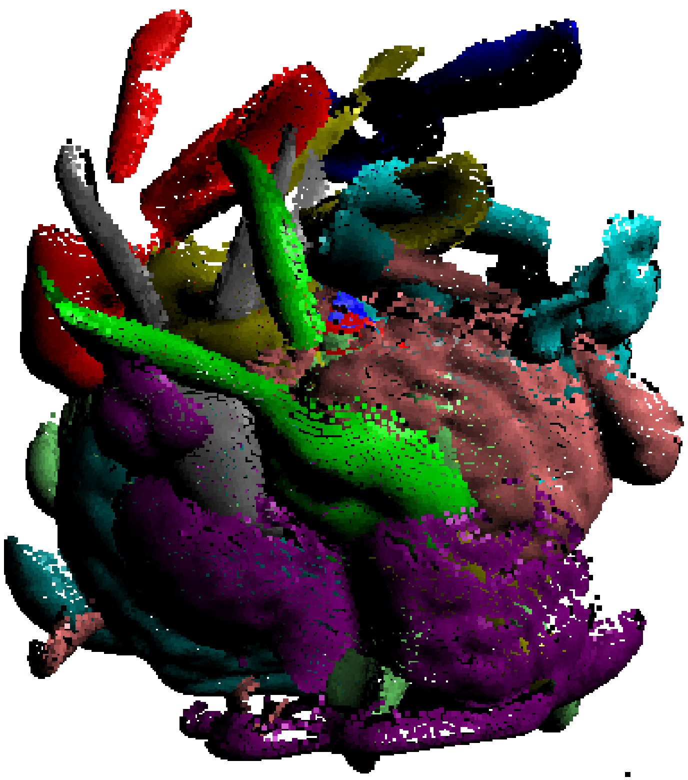
Scan Registration



[data set: Stanford 3D Scanning Repository]

Feature Tracking

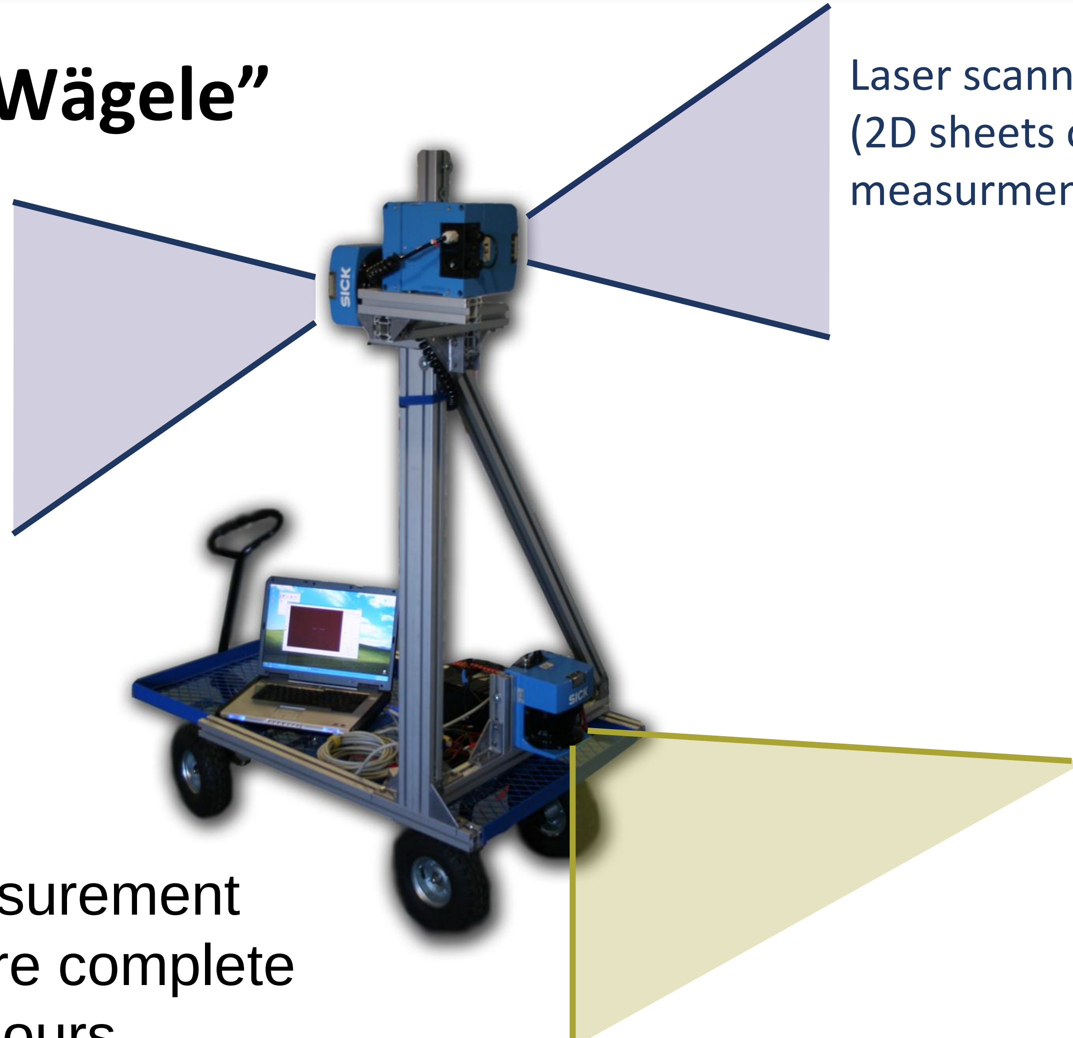
Fully Automatic:



[Implementation: Martin Bokeloh (Diploma thesis)]

Scanning the World....

Example: The "Wägele"



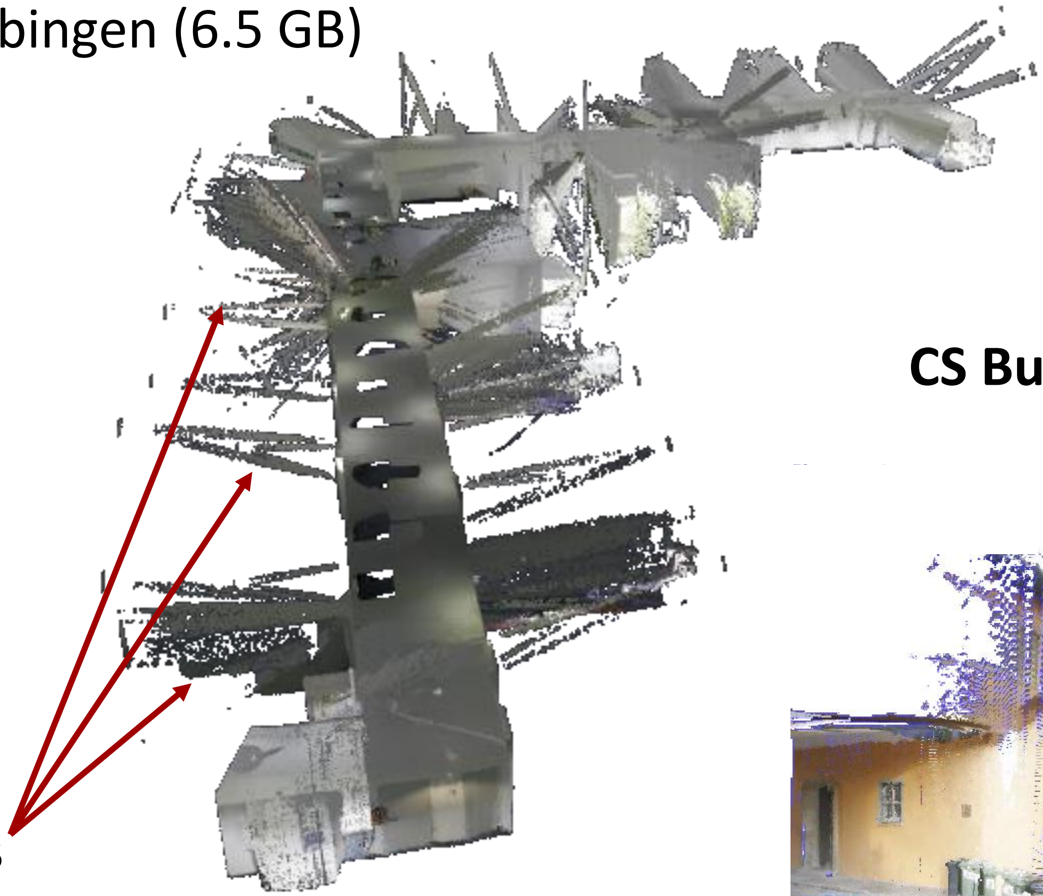
Laser scanners
(2D sheets of distance
measurments)

[Biber et al. 2005]

A pull-through measurement device – can acquire complete buildings in a few hours

This is what you get...

Corridor – CS Building
University of Tübingen (6.5 GB)

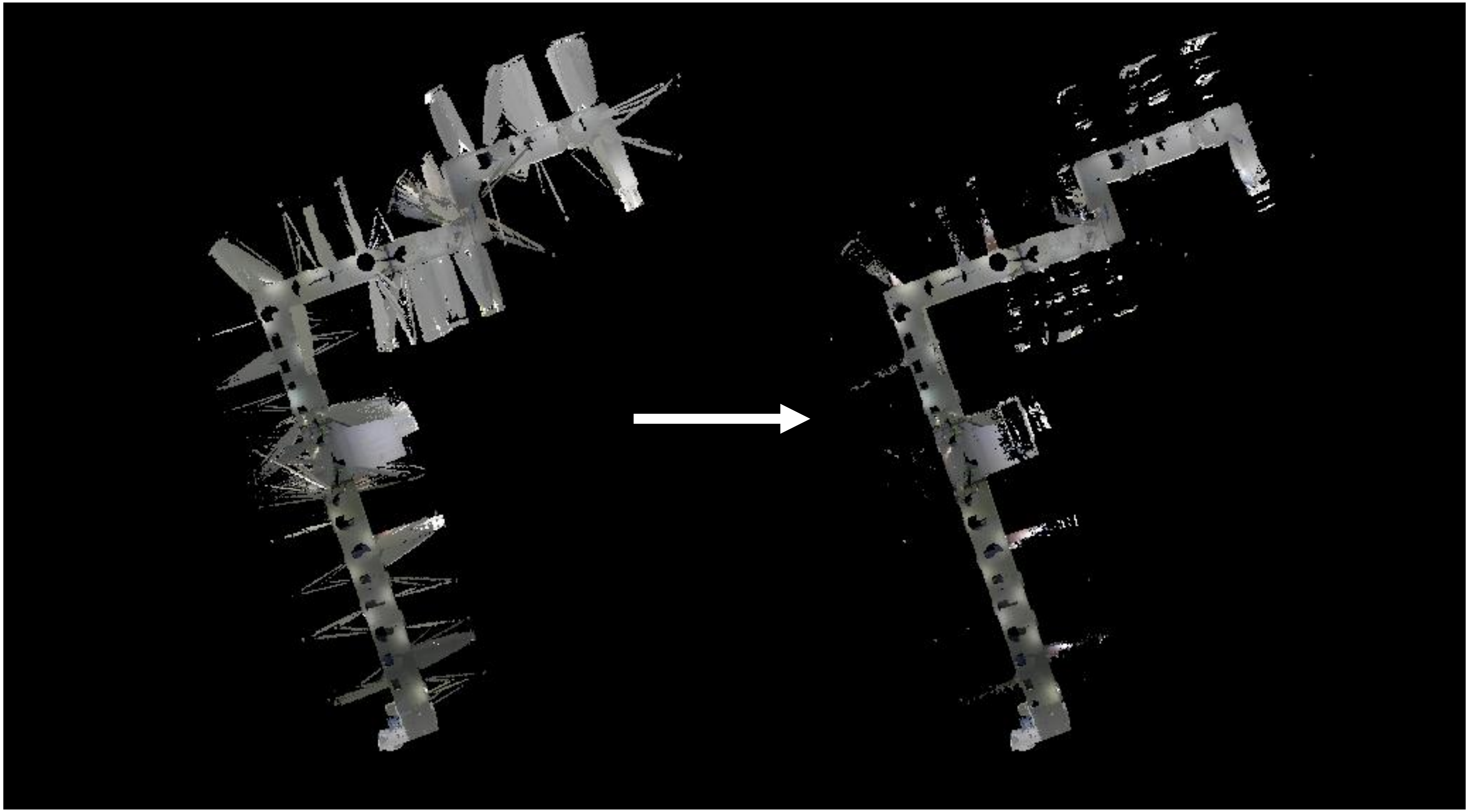


...lots of artifacts
(the scanner does not really like windows)

CS Building, Outside
(nicer colors...)

Automatic Processing

Example: Automatic Outlier Removal



Think Big

More Problems:

- Occluded areas, shiny / transparent objects
  $\Rightarrow$ holes (lots of holes, actually)
- Huge amounts of data (really huge)

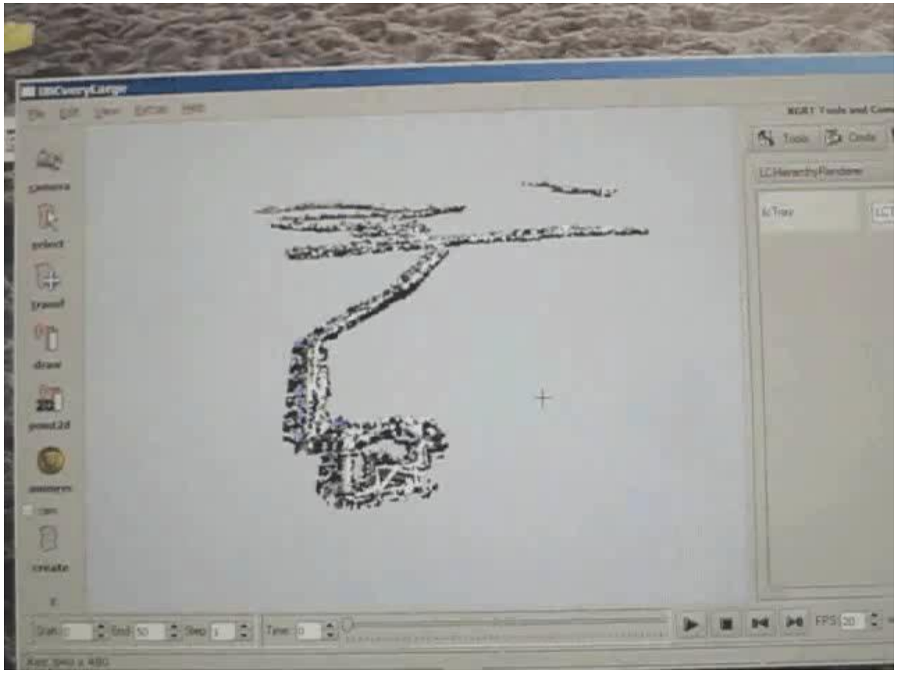
City Scanning

- There are big companies trying to scan large areas
- Think Google Earth in full resolution
- How about a virtual online walk through
  New York, *Tokyo*, *Saarbrücken*?
- Lots of open research problems to get there

[data set: Institute for Cartography, Leibnitz University Hannover]

HUGE Data Sets

The Largest Data Set I have On My Hard-Drive...



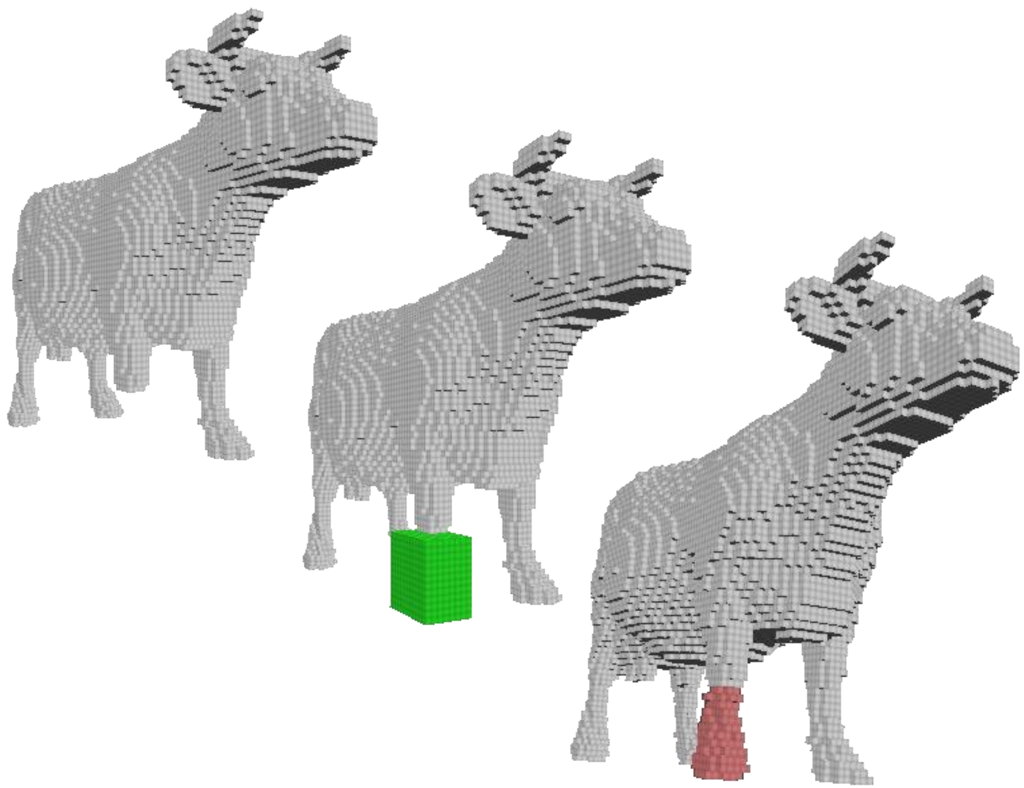
Data set: Outdoor Scan (structure from video) of a part of the UNC campus ($2.2 \cdot 10^9$ pts / 63.5 GB), courtesy of J.-M. Frahm, University of North Carolina

Hole Filling

Wei-Levoy Texture Synthesis Algorithm:

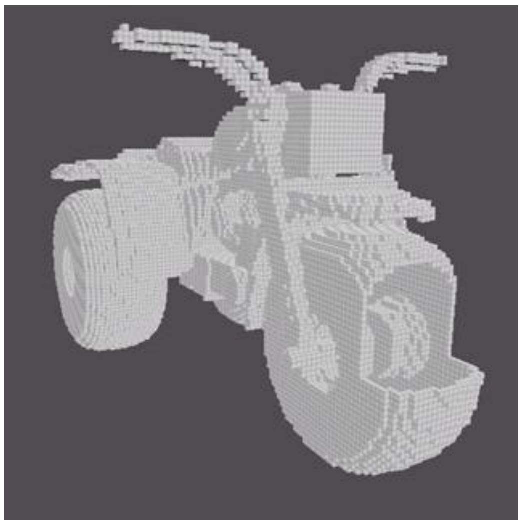
[Implementation: Alexander Berner (Diploma thesis)]

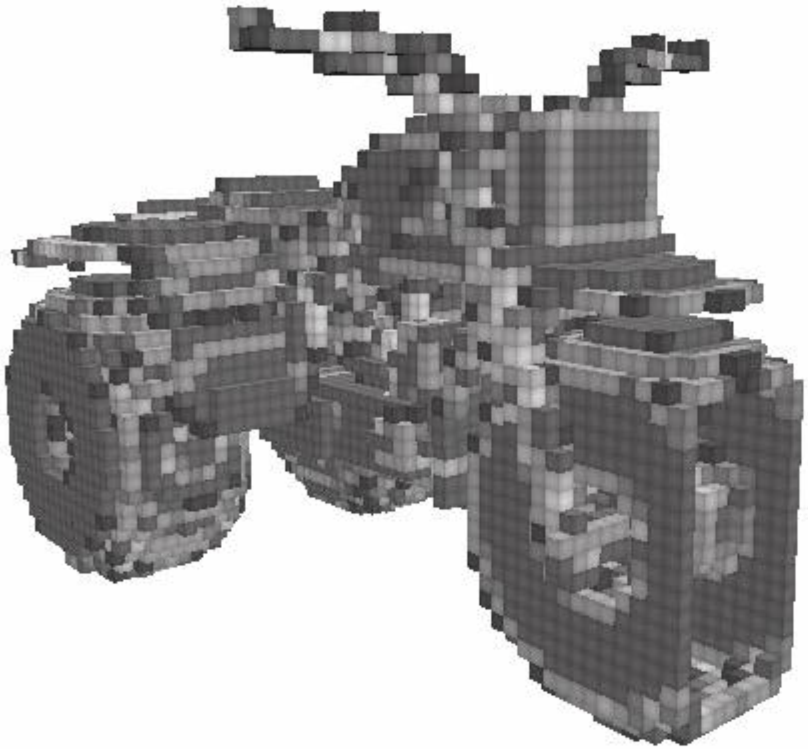
Filling Holes



[implementation: Alexander Berner (Diploma thesis)]

Filling Holes



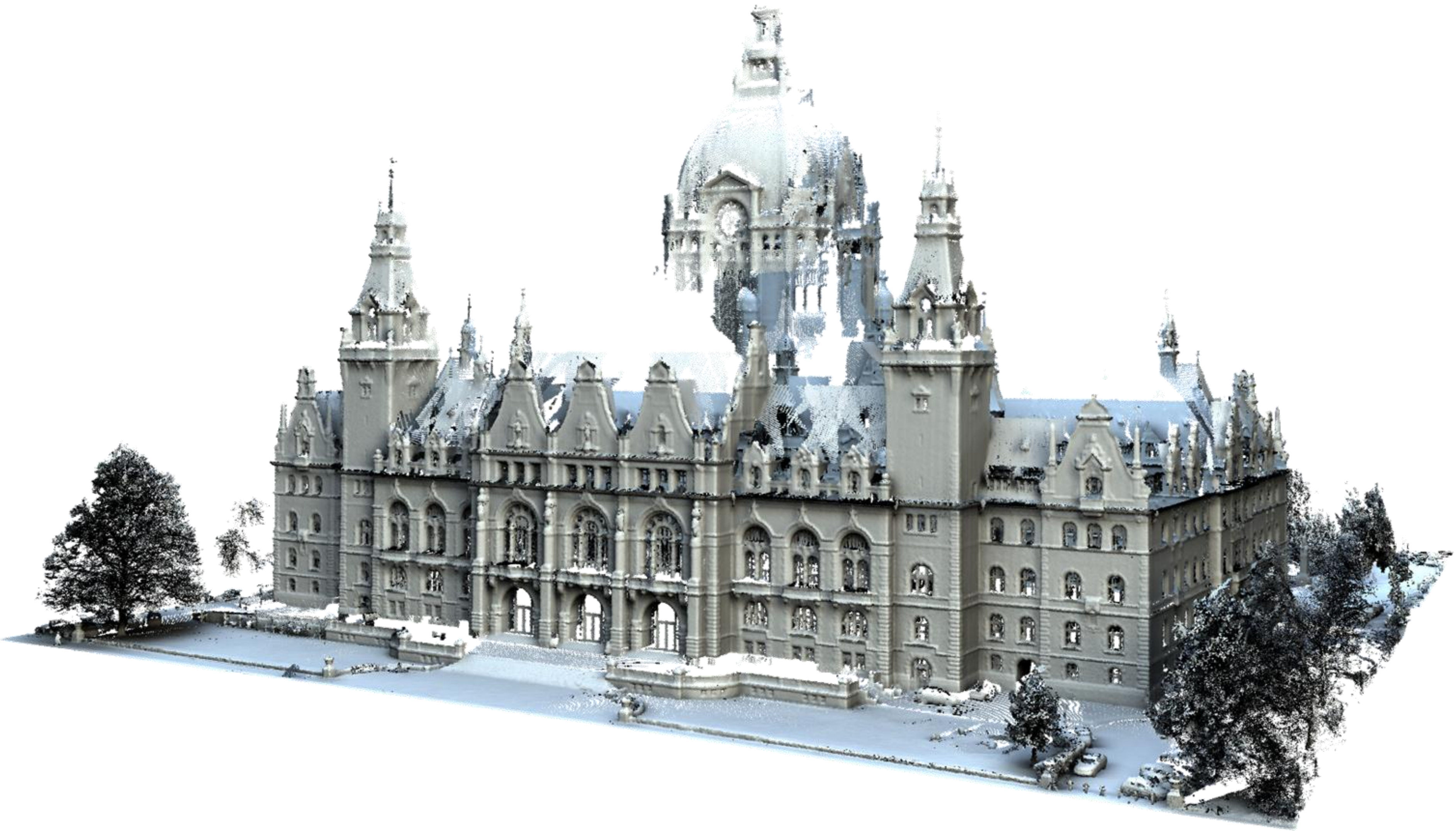


[implementation: Alexander Berner (Diploma thesis)]

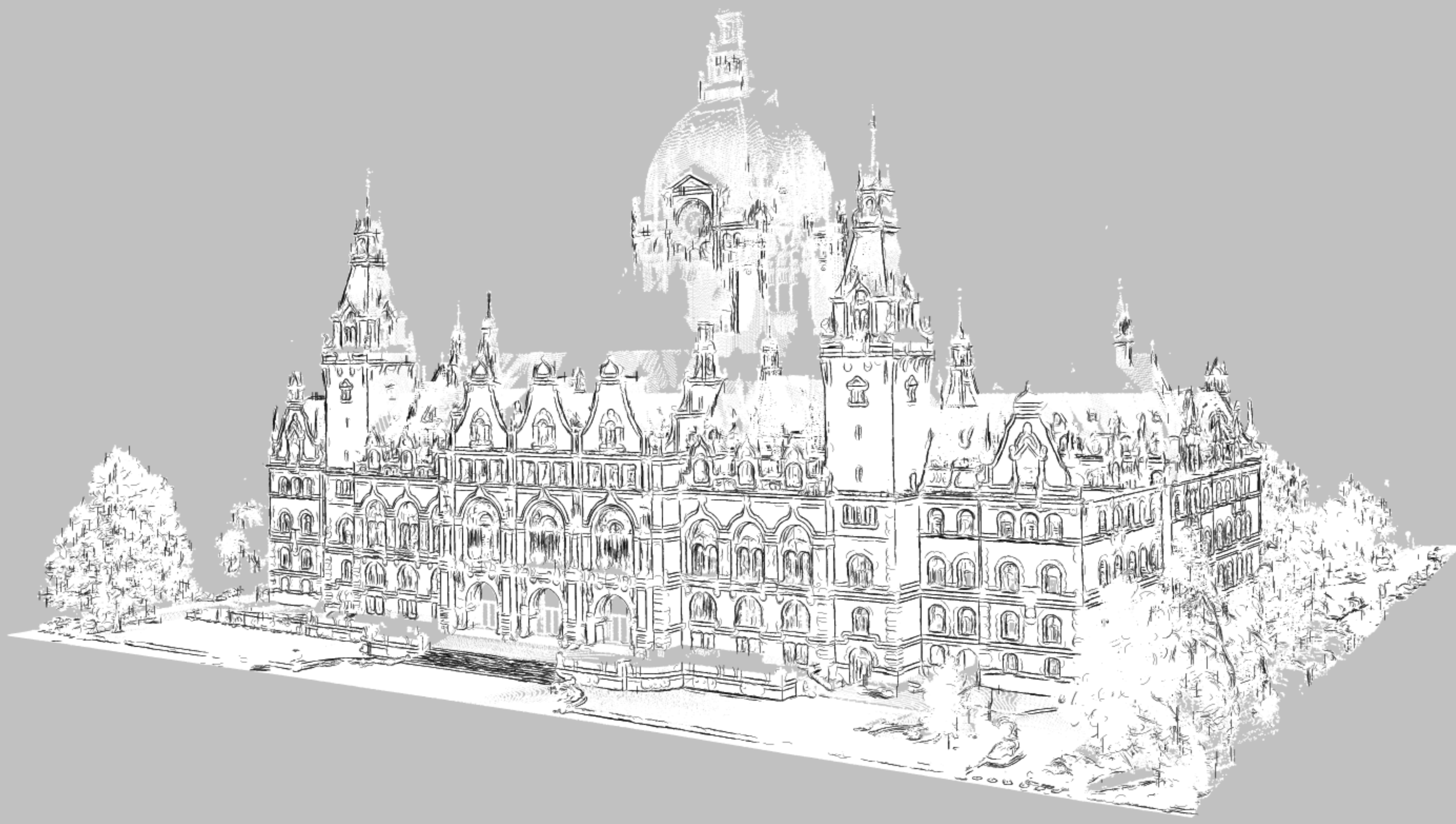
Symmetry Detection

[data set: M. Wacker, HTW Dresden]

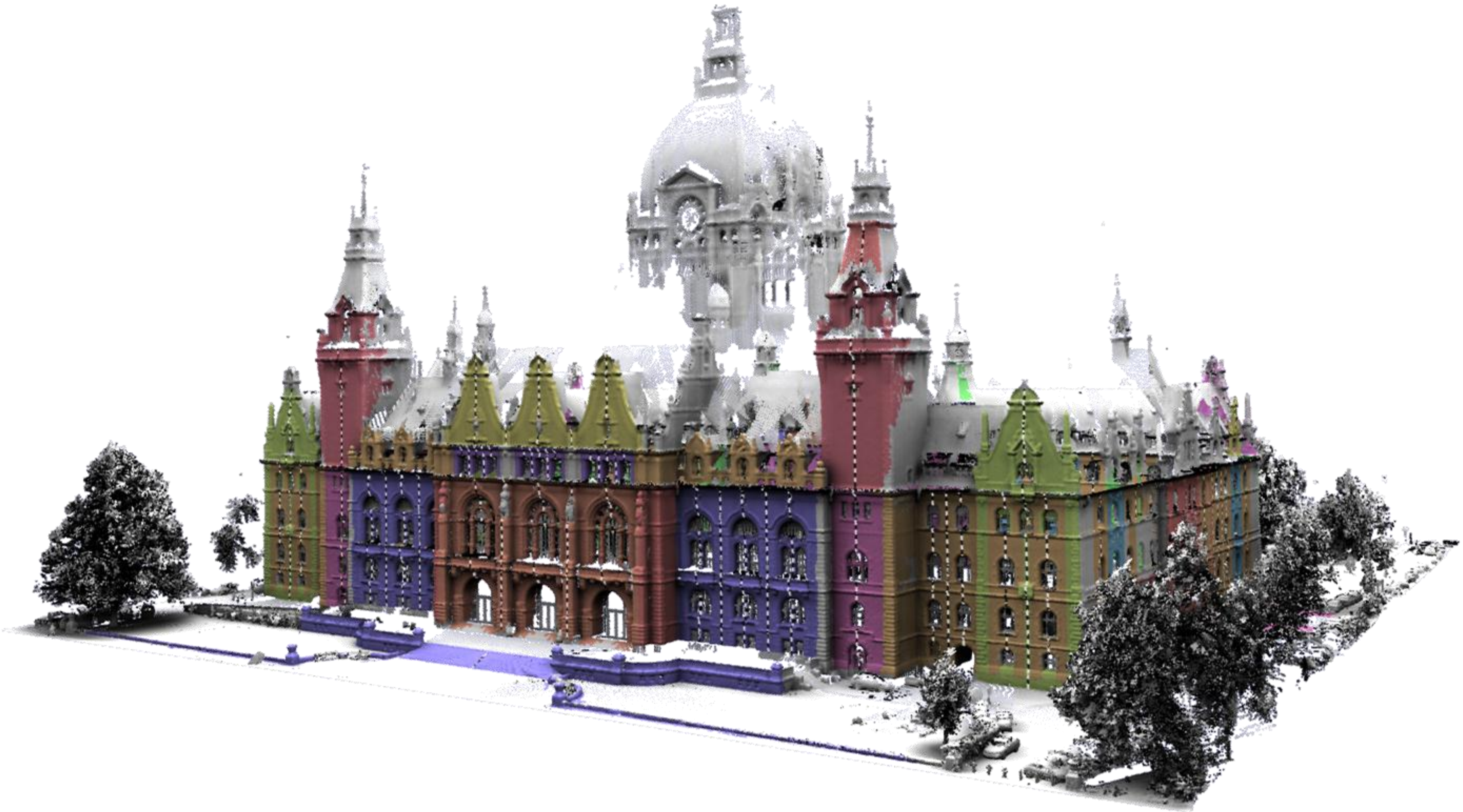
Results



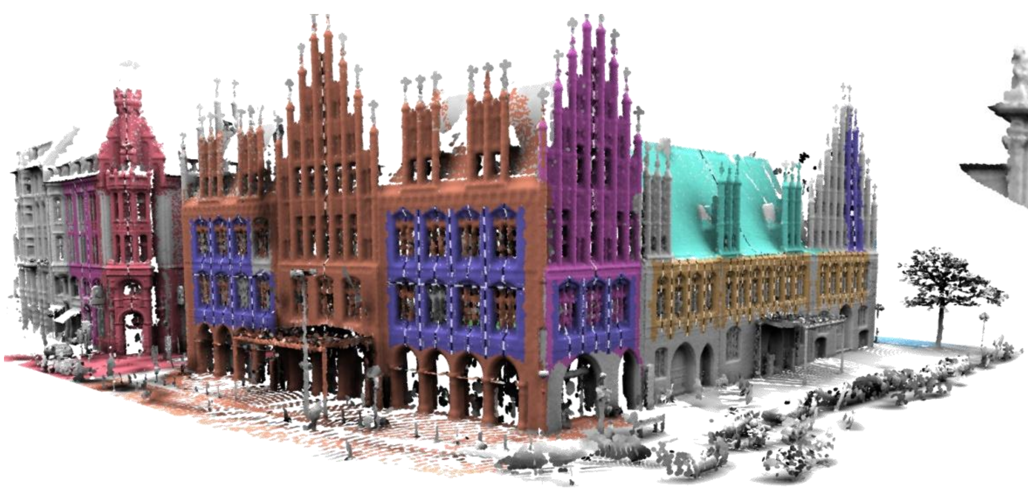
Results

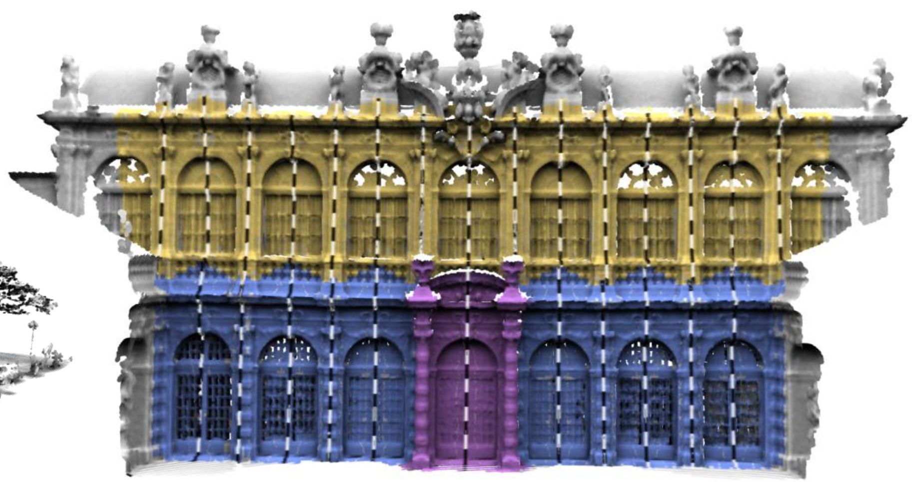


Results



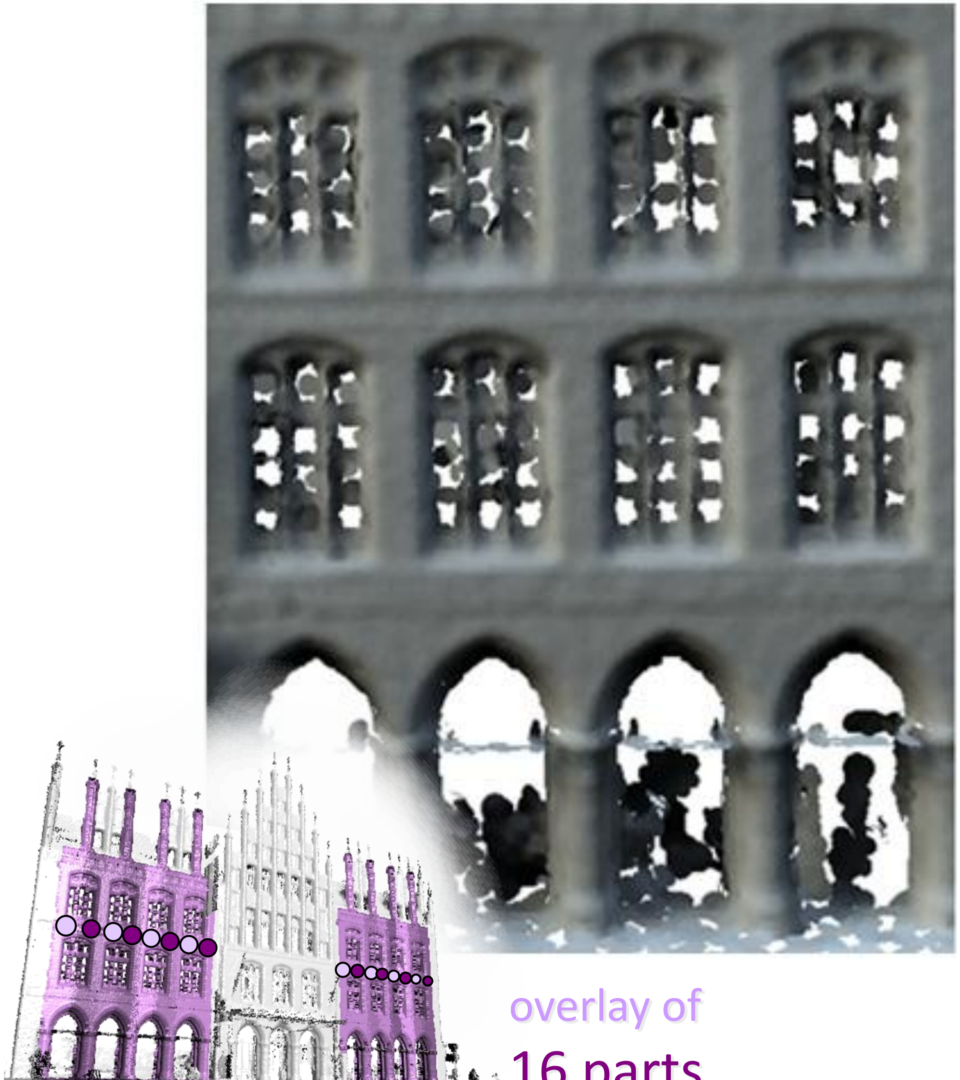
Line Feature Matching





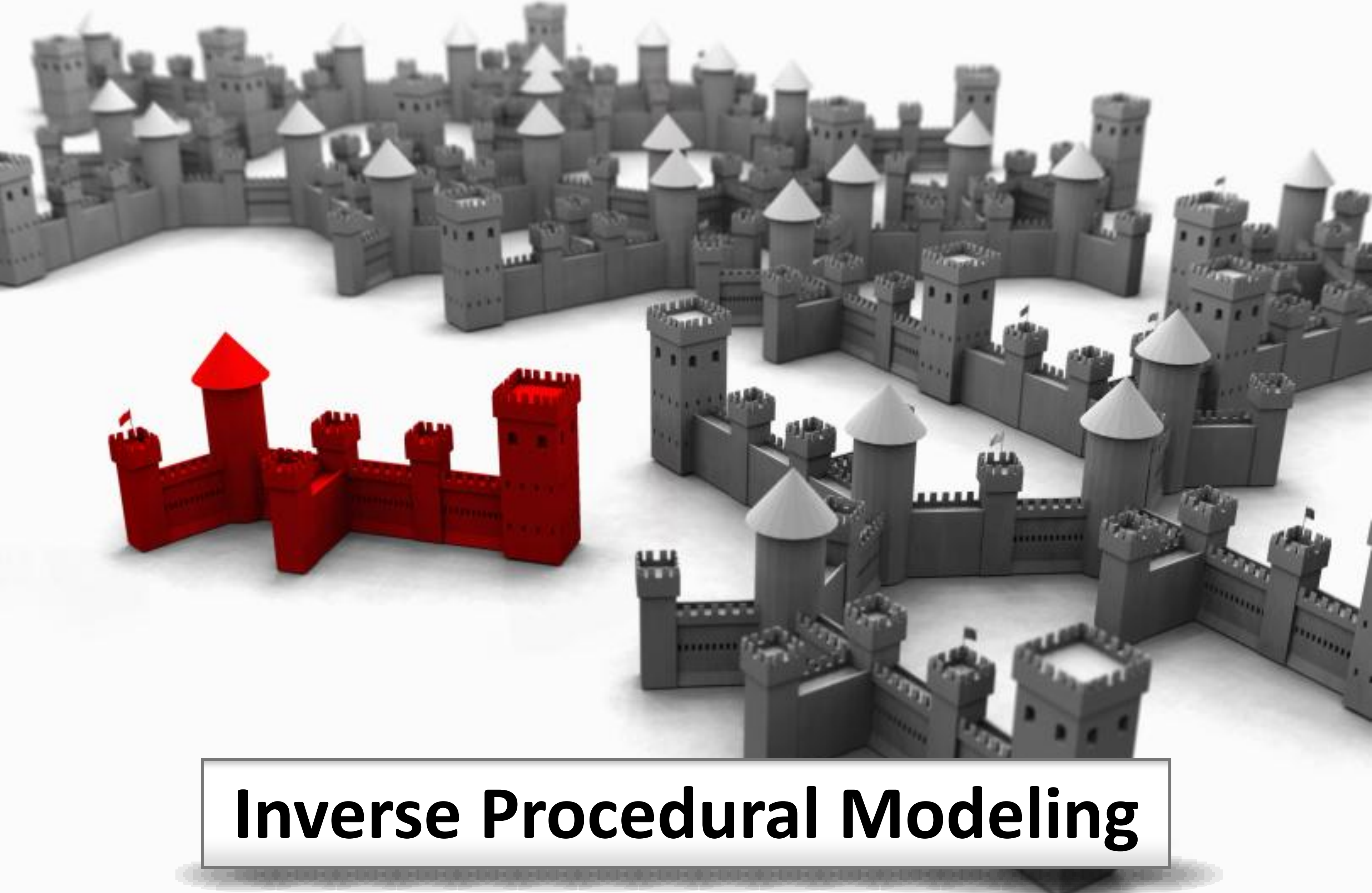
[data sets: Kartographisches Institut, Universität Hannover / M. Wacker, HTW Dresden]

Reconstruction by Symmetry



overlay of
16 parts

[data sets: Kartographisches Institut, Universität Hannover]

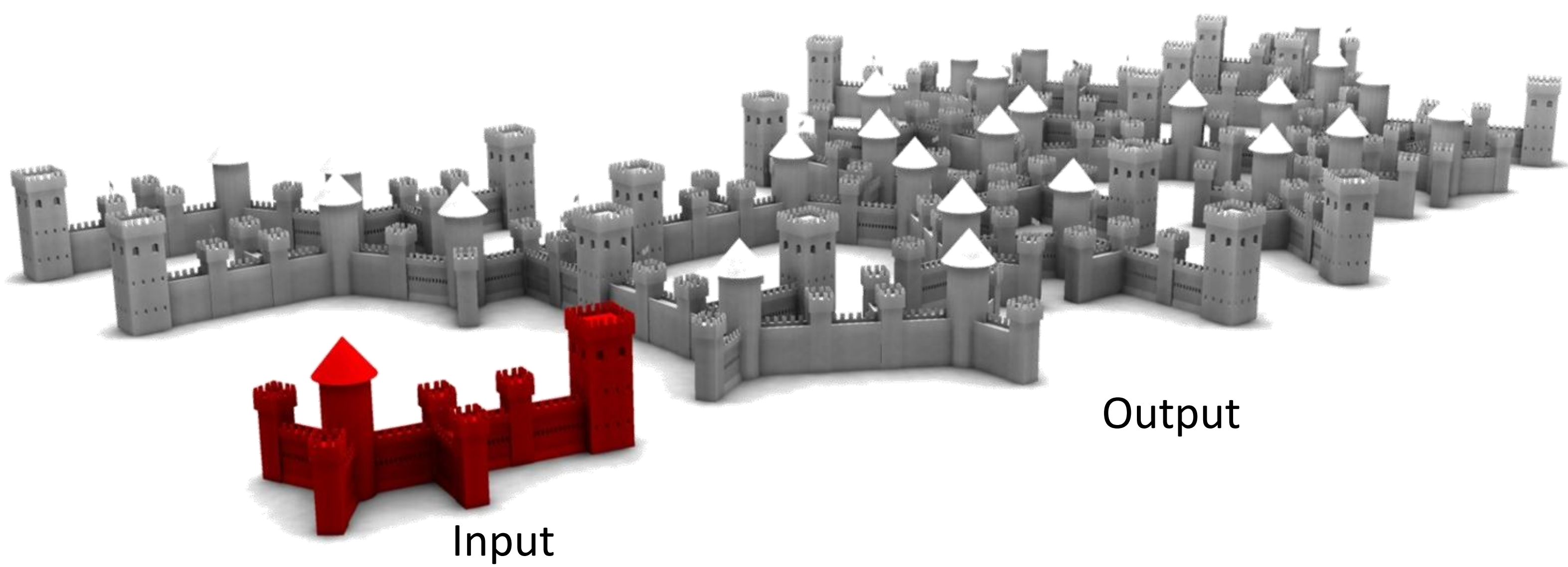


Inverse Procedural Modeling

Overview

Our approach

- Take *existing* model
- Analyse shape structure
- Derive shape modification rules



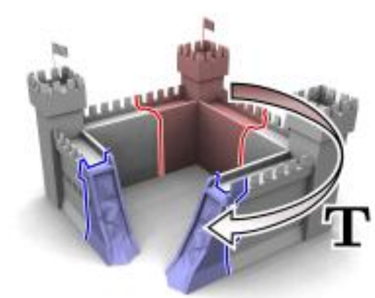
Technique Overview

Conceptual Steps:

- Symmetry detection
- Finding *docking sites* and *dockers*
- Combine into a *shape grammar*

Input

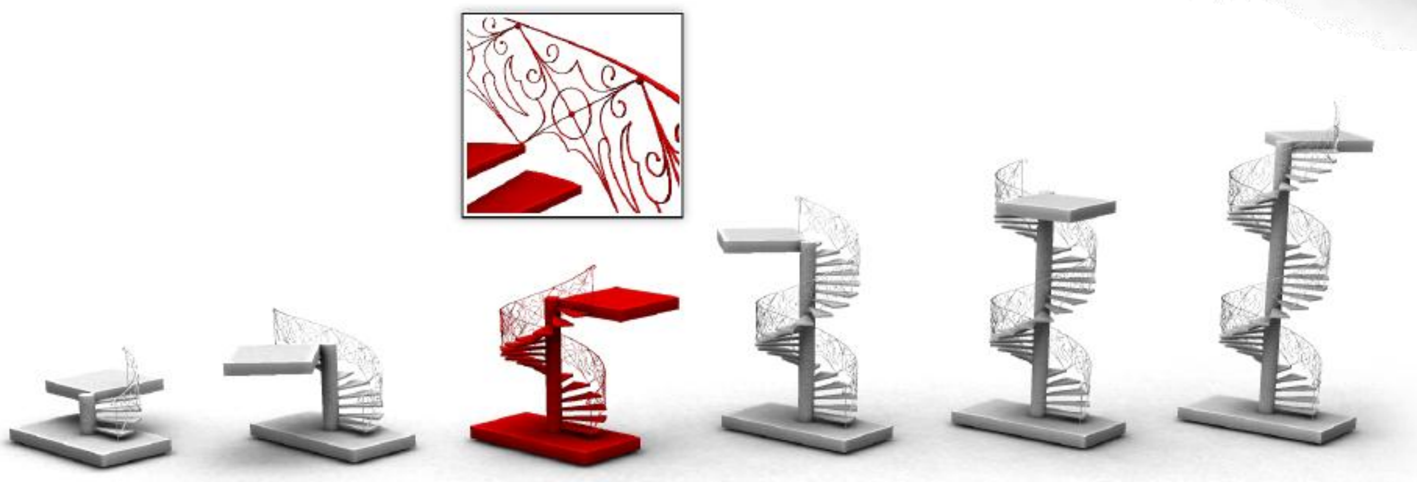
Symmetry



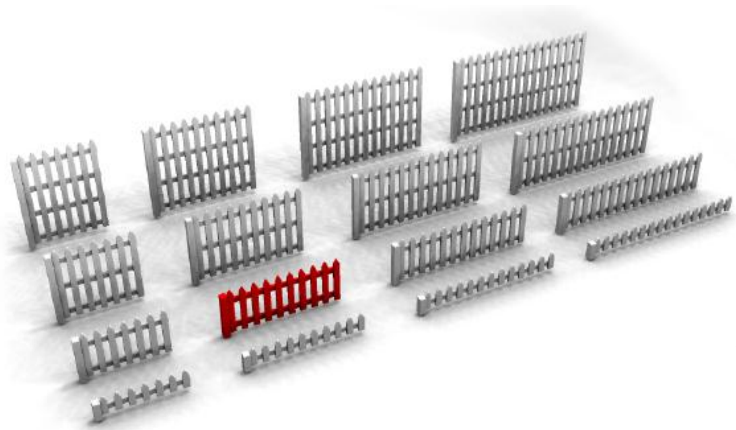
Docking site

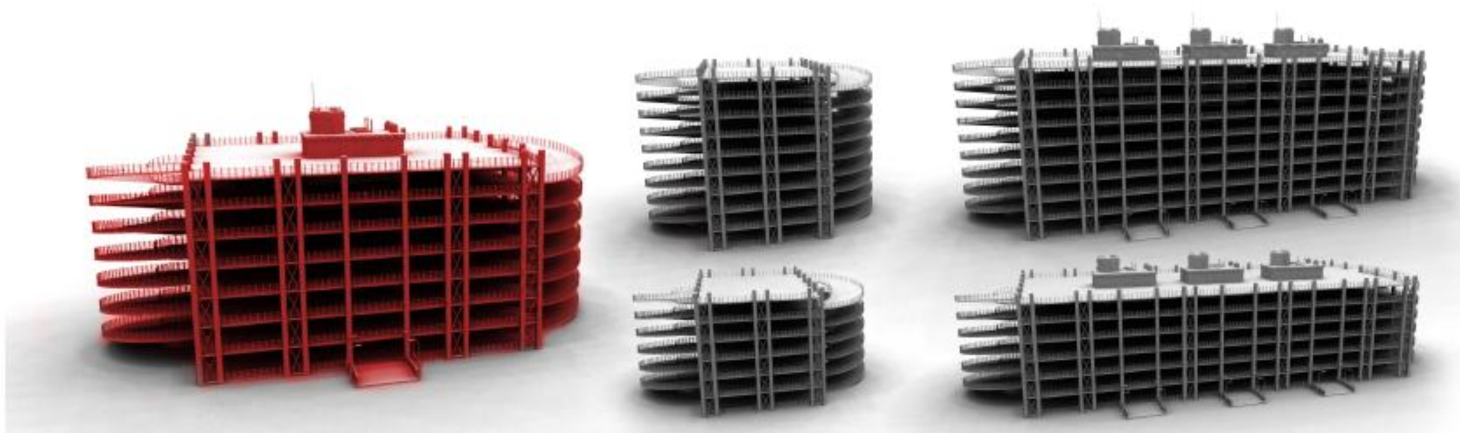
Result

Results



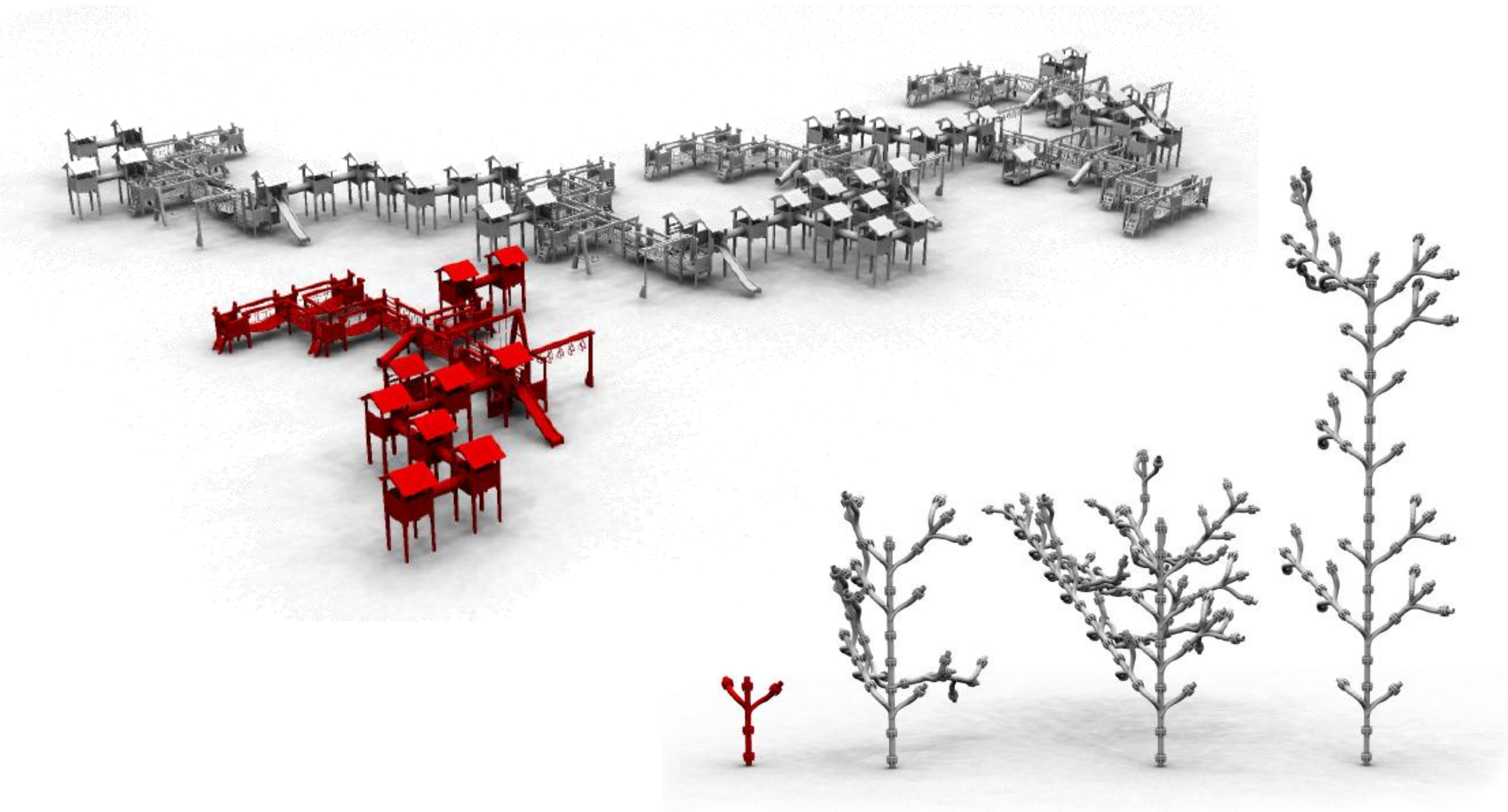
Results



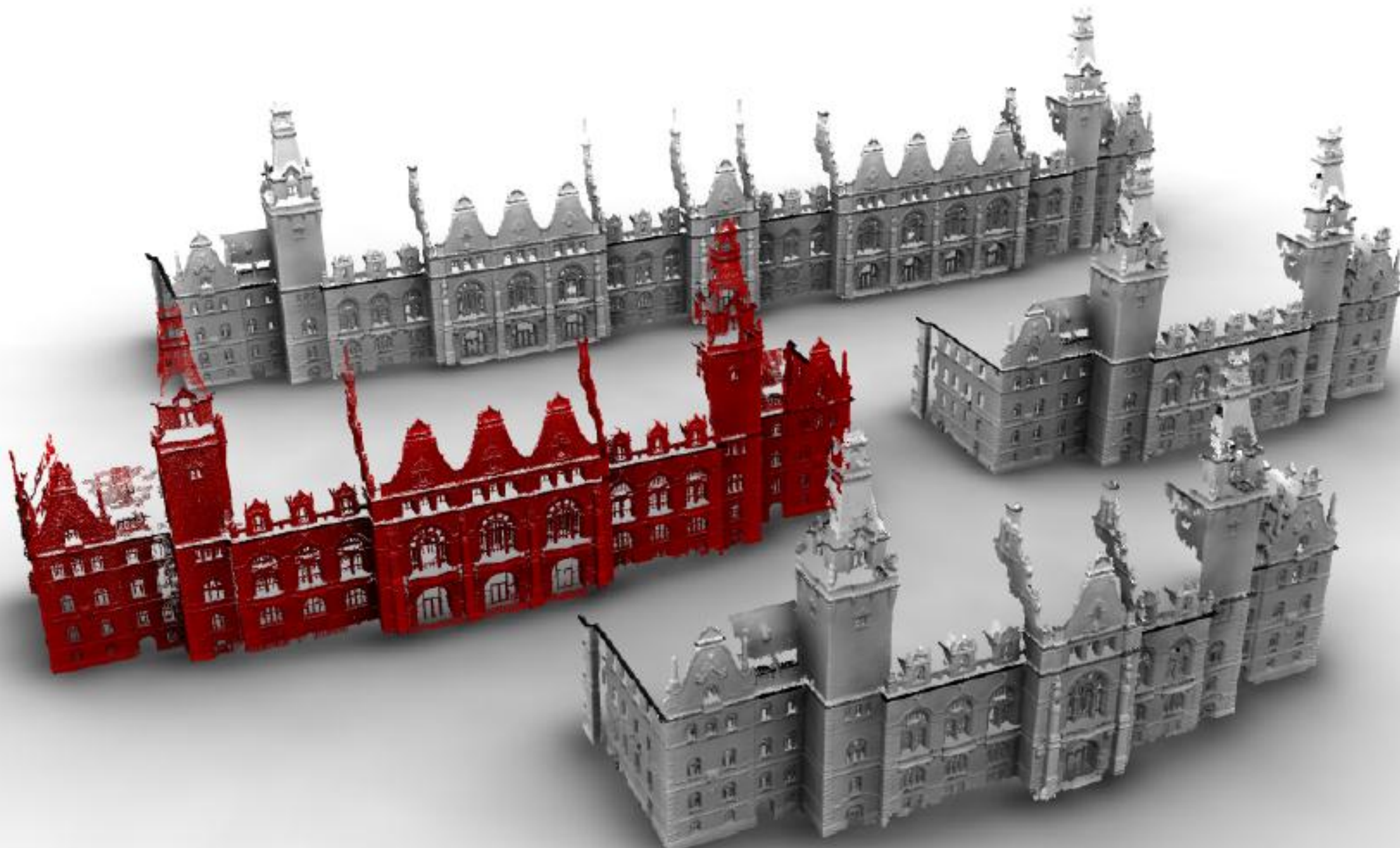


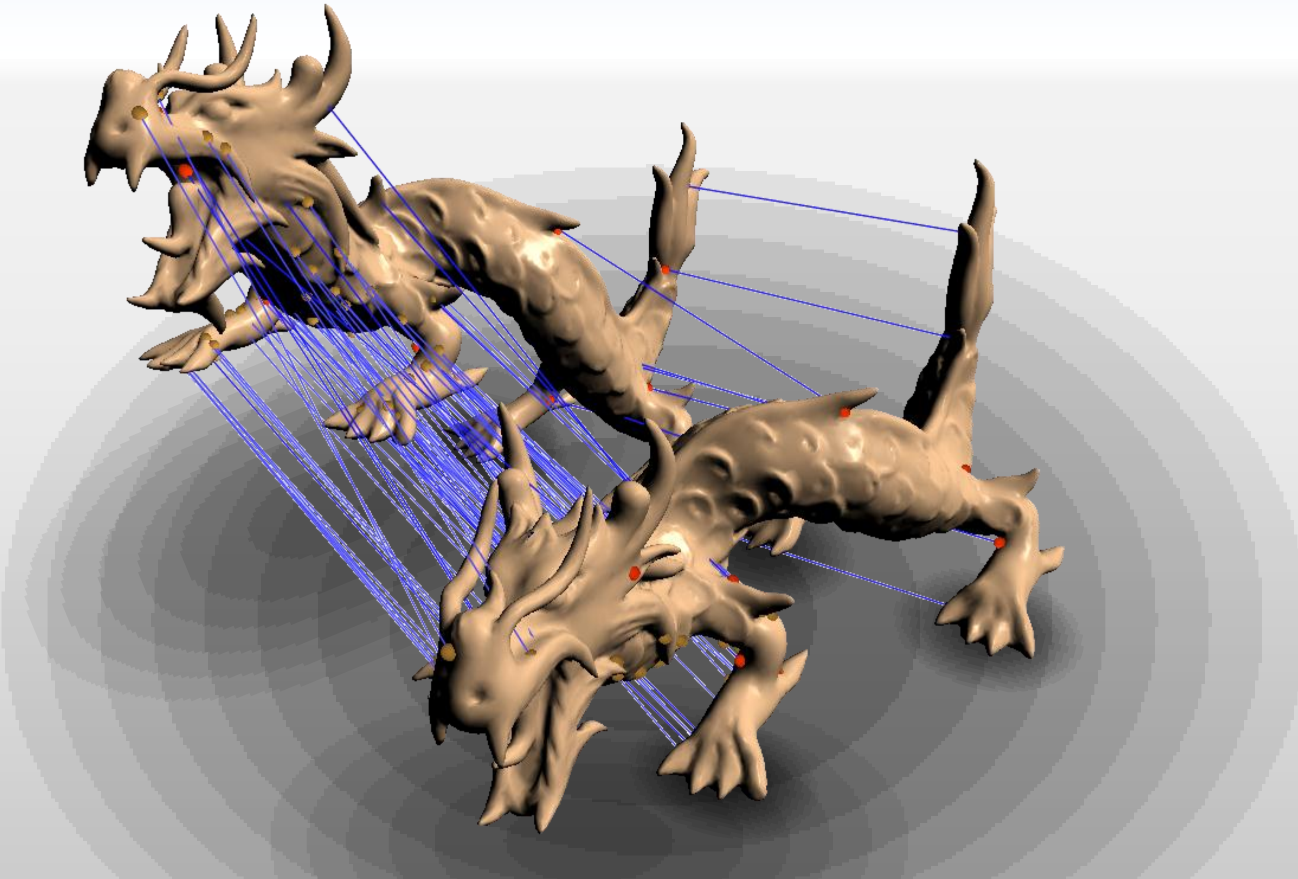
~500.000 triangles

Results



Results





Deformable Shape Matching

[data set: Stanford 3D Scanning Repository]

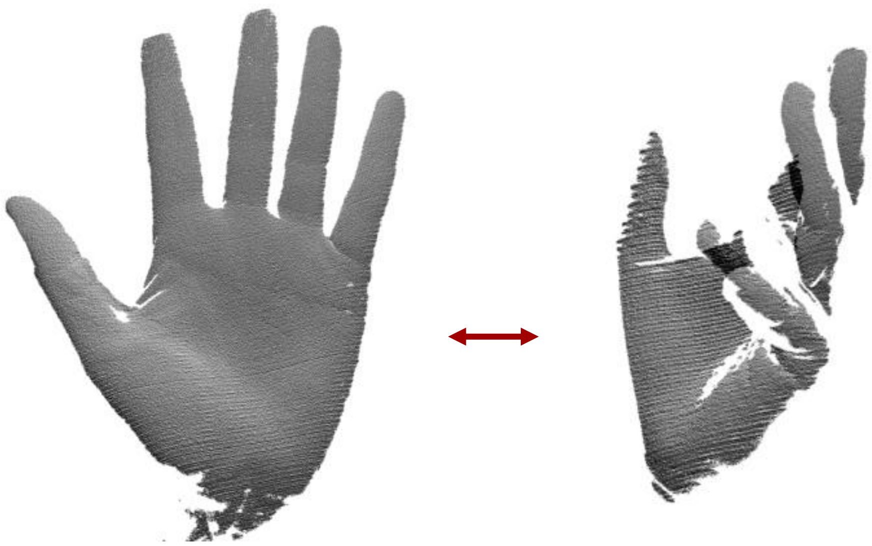
Problem Statement

Deformable Matching

- Two shapes: original, deformed
- How to establish correspondences?
- Looking for global optimum
  - Arbitrary pose

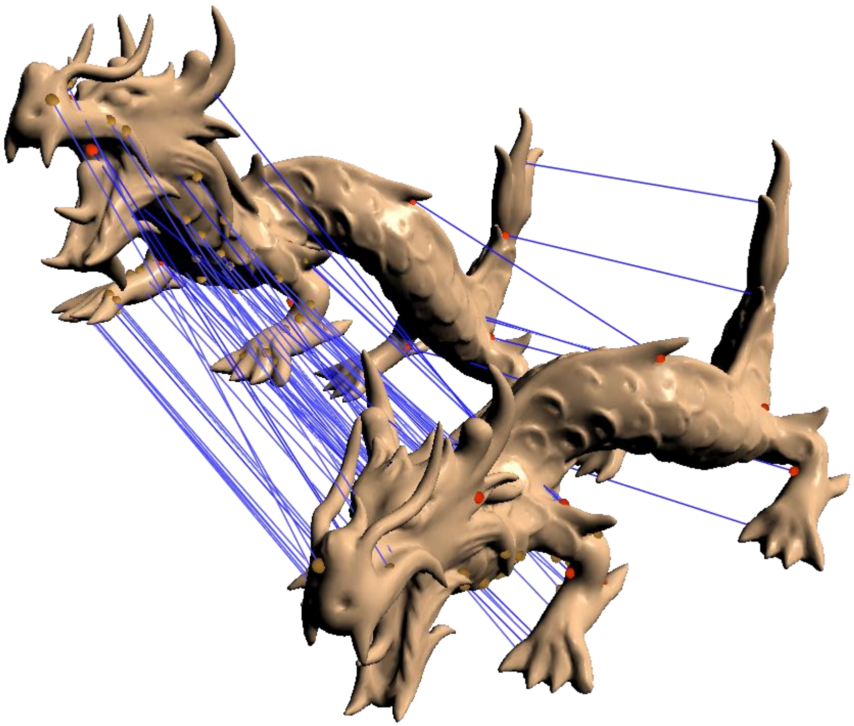
Assumption

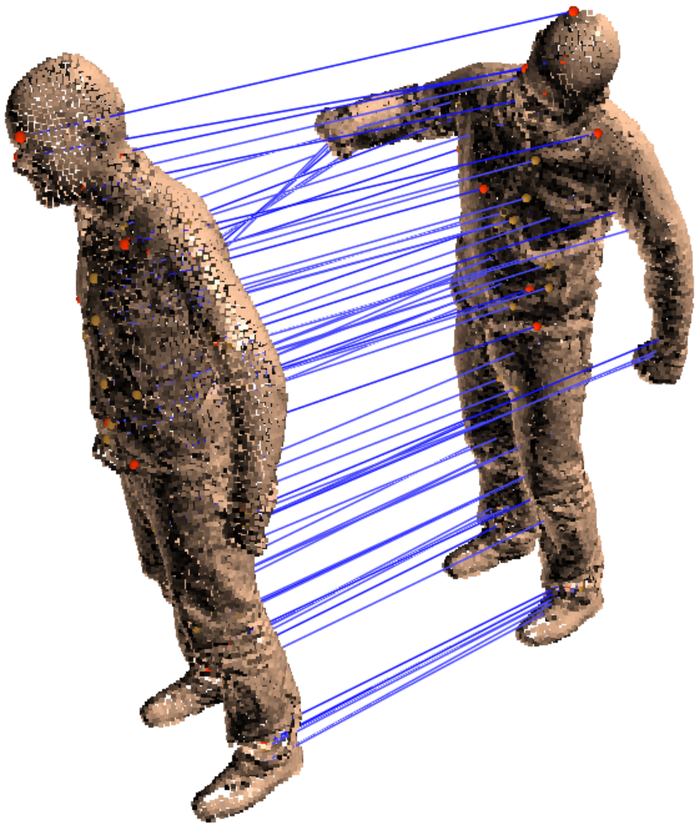
- Approximately isometric deformation



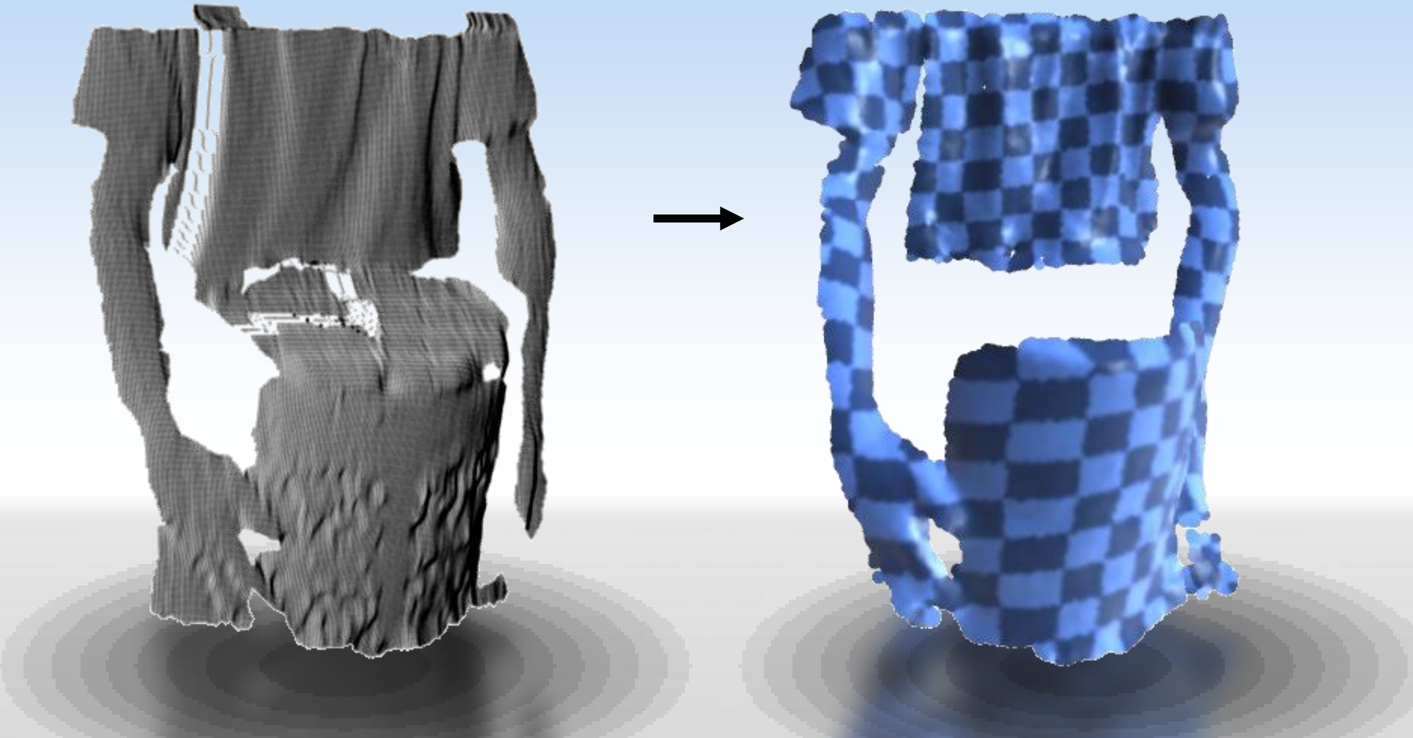
[data set: S. König, TU Dresden]

Results





[data sets: Stanford 3D Scanning Repository / Carsten Stoll]



Animation Reconstruction

[data set: P. Phong, Stanford University]

Scanning Moving Geometry

Real-time 3D scanners:

- Acquire geometry at video rates
- Capture 3D movies: “performance capture”
- Technique still immature, but very interesting applications, in particular special effects for movies

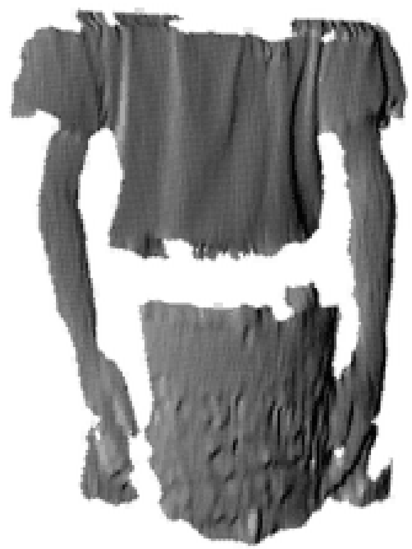
[Davis et al. 2003]

Real-Time 3D Scanners

space-time stereo

courtesy of James Davis

University of California at Santa Cruz



color-coded structured light

courtesy of Phil Fong

Stanford University

high-speed structured light

courtesy of Stefan Gumhold

TU Dresden

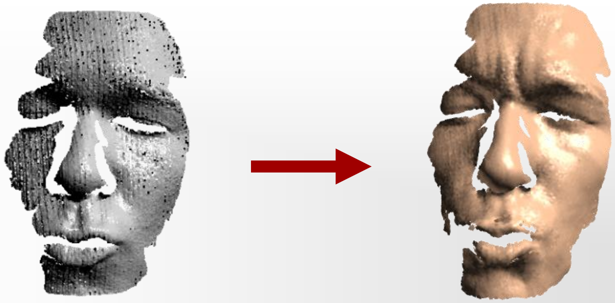
Reconstruction

Dynamic geometry reconstruction

- Hole filling
- Remove noise and outliers
- Establish correspondences
  - Need to know where every point on the object goes to over time
  - Simplifies further editing

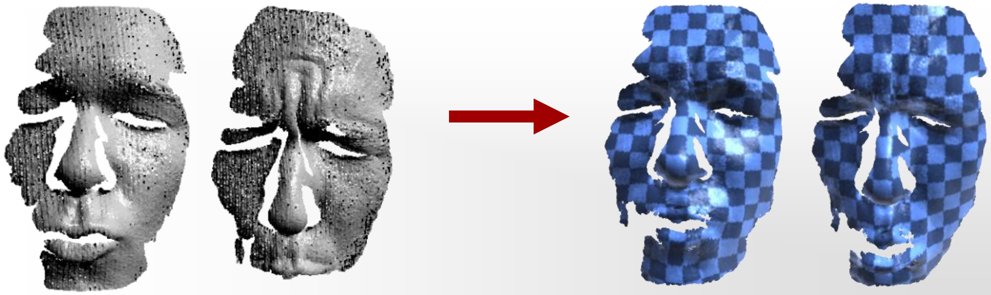
Animation Reconstruction

Remove noise, outliers

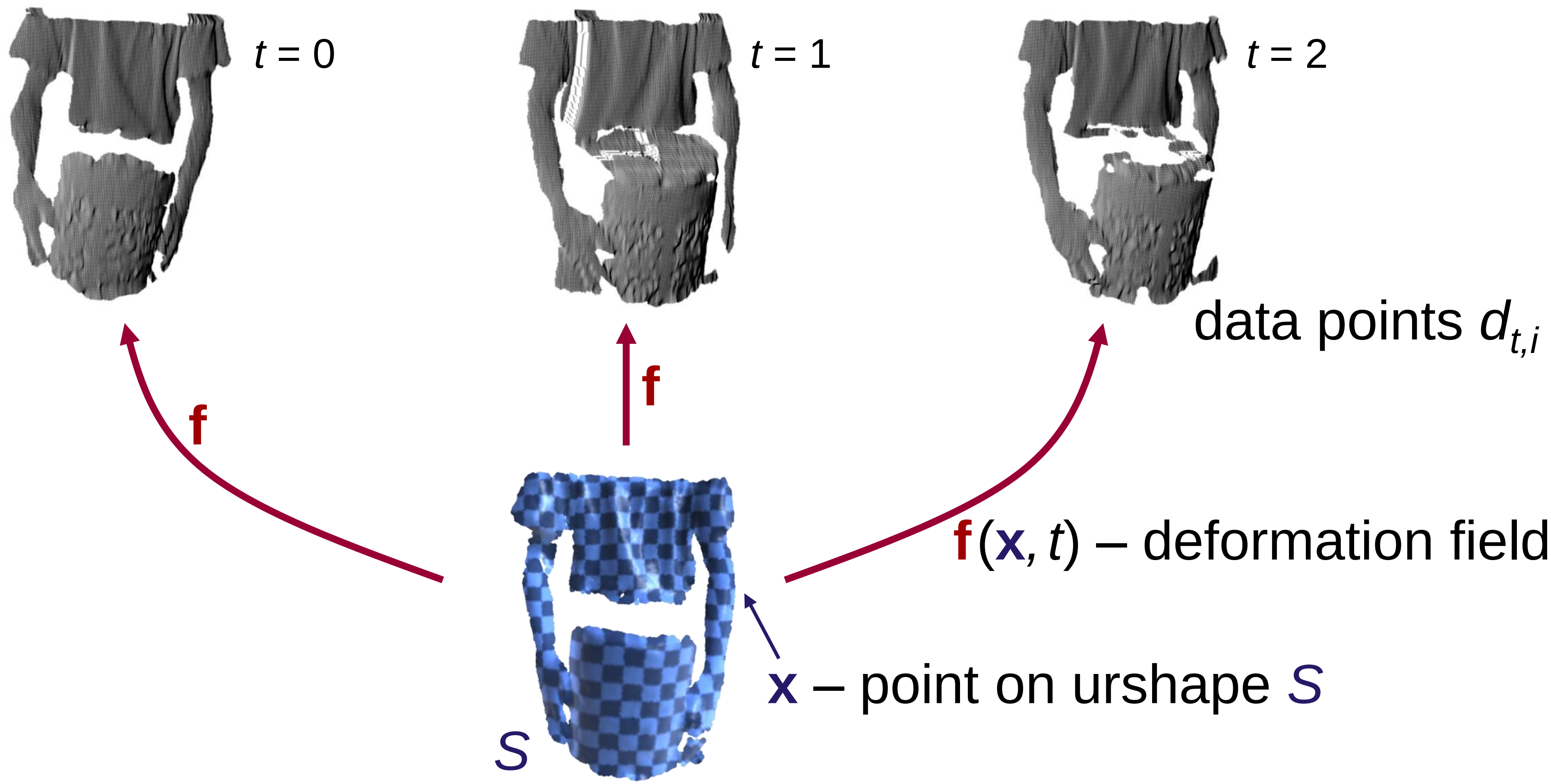


Fill-in holes
(from all frames)

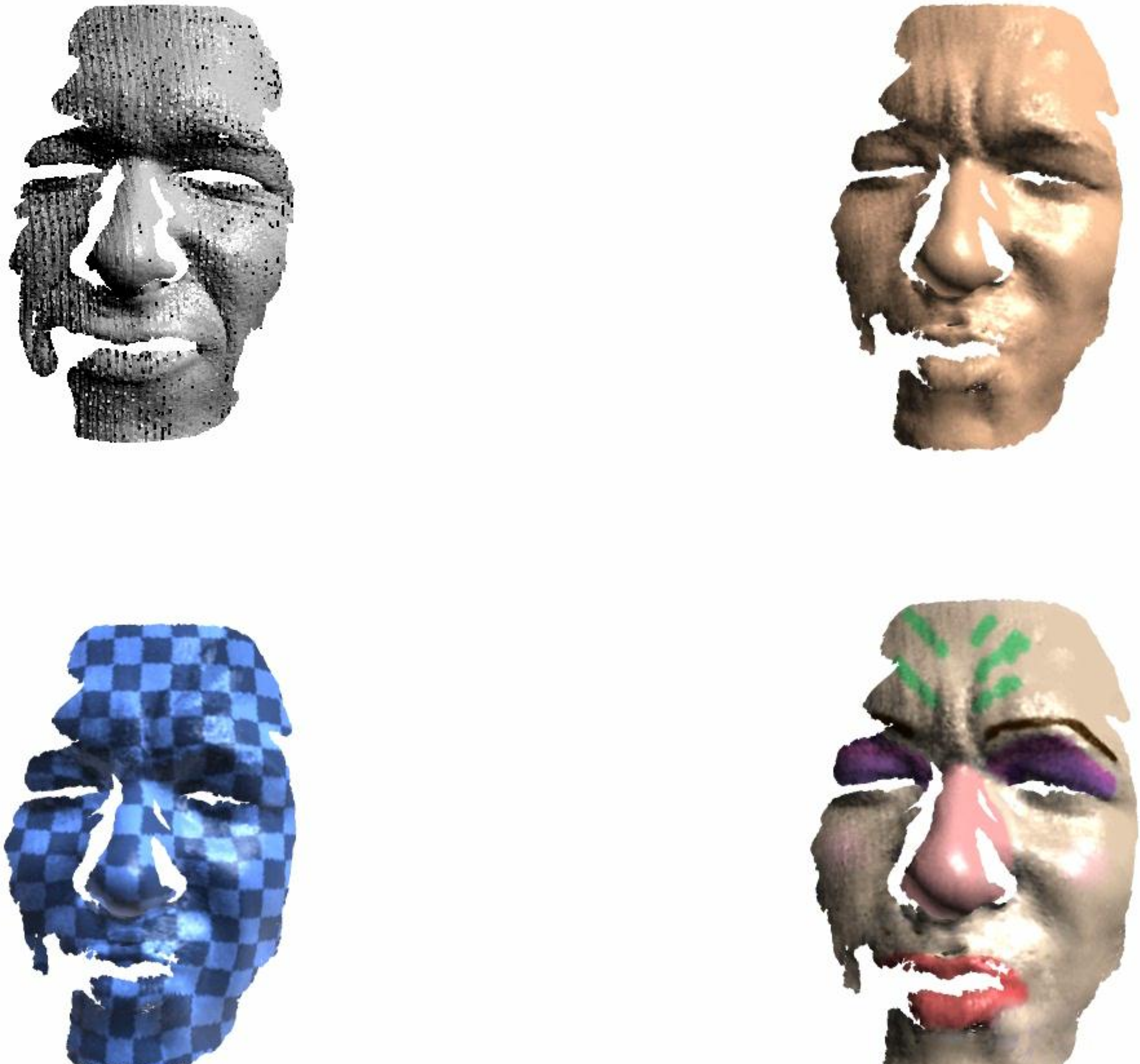
Dense correspondences



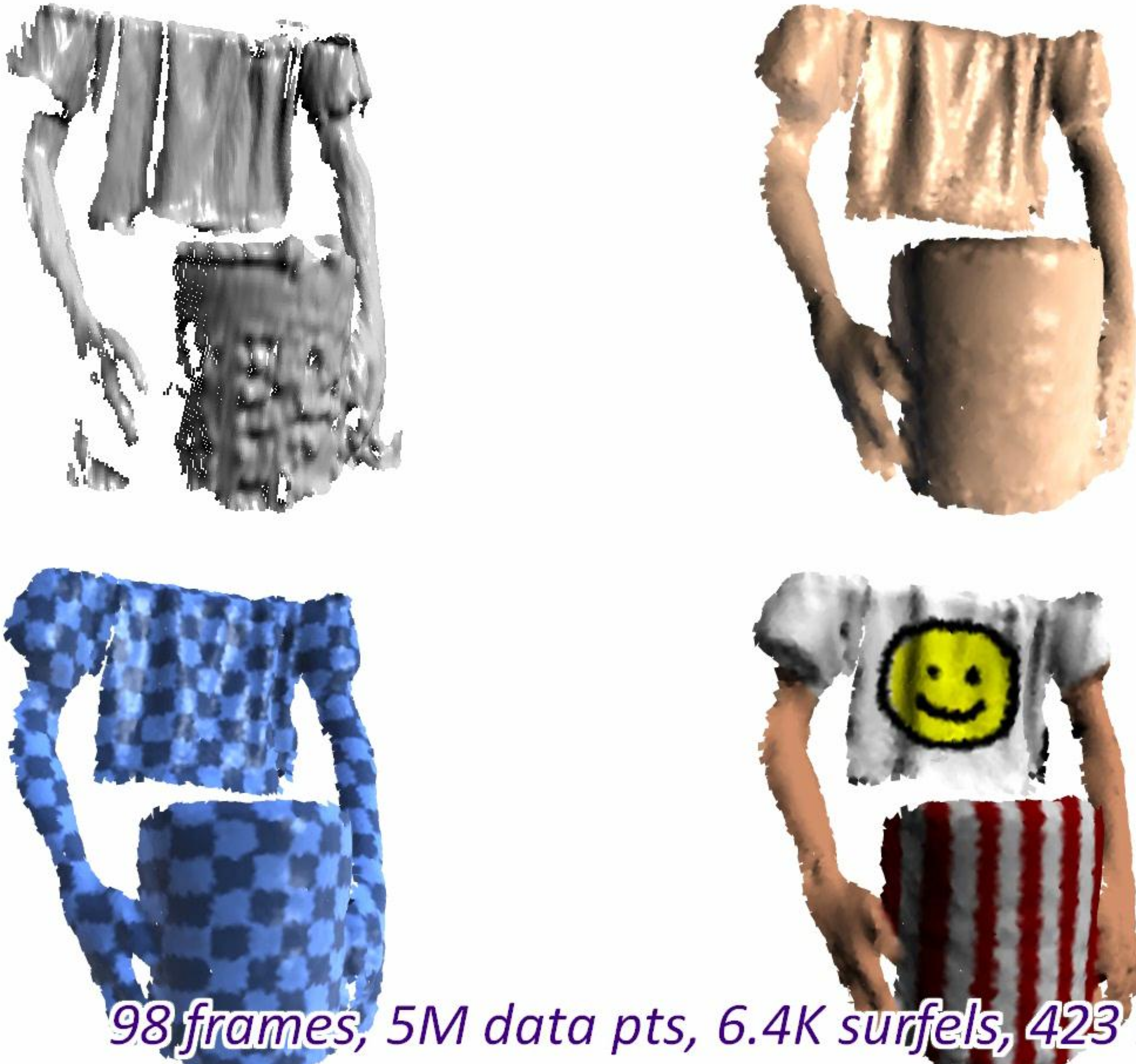
Factorization



[data set courtesy of P. Phong, Stanford University]



79 frames, 24M data pts, 21K surfels, 315 nodes



98 frames, 5M data pts, 6.4K surfels, 423 nodes

Overview

Topics

Overview: Geometric Modeling 2010

Mathematical Background (Recap)

- Linear algebra: vector spaces, function spaces, quadrics
- Analysis: multi-dim. calculus, differential geometry
- Numerics: quadratic and non-linear optimization

Geometric Modeling

- Smooth curves: polynomial interpolation & approximation, Bezier curves, B-Splines, NURBS
- Smooth surfaces: spline surfaces, implicit functions, variational modeling
- Meshes: meshes, multi-resolution, subdivision

Overview: Geometric Modeling 2010

Geometry Processing

- 3D Scanning: Overview
- Registration: ICP, NDT
- Surface Reconstruction: smoothing, topology reconstruction, moving least-squares
- Editing: free-form deformation

Preliminary List:

- Topics might change
- Not presented strictly in this order

Topics Overview

Current List of Topics (subject to changes):

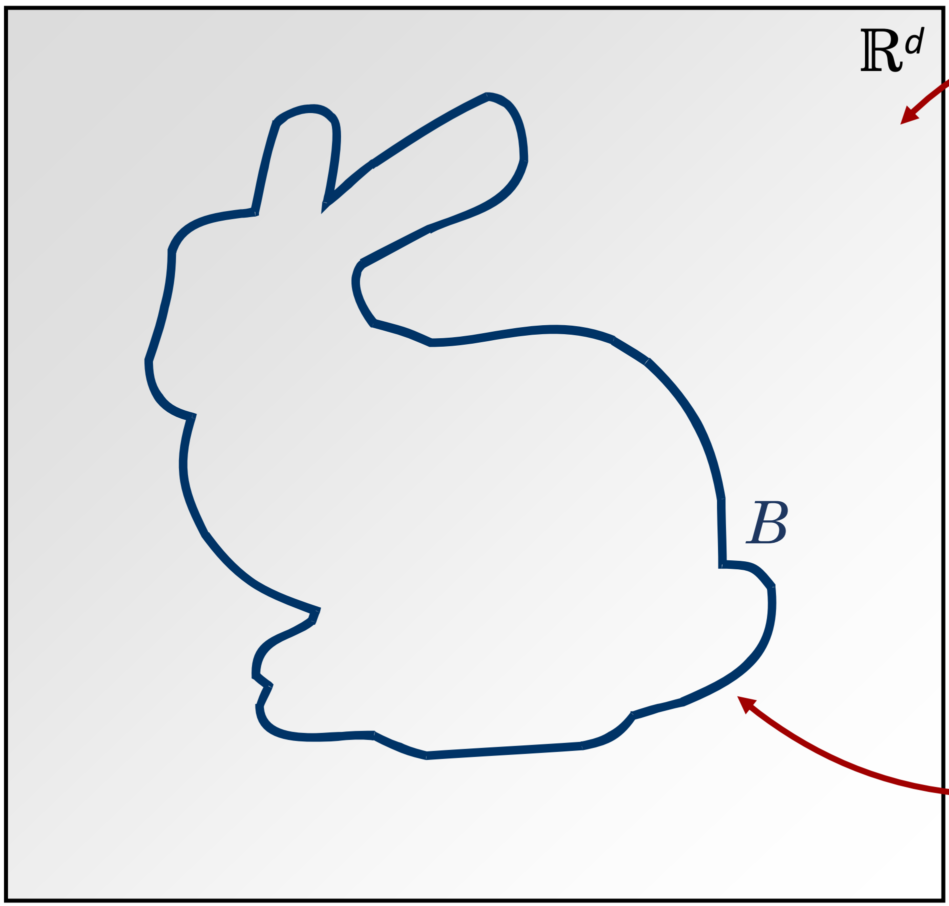
- Math Background:
  - Linear Algebra, Analysis, Differential Geometry, Numerics, Topology
- Interpolation and Approximation
- Spline Curves
- Blossoming and Polar Forms
- Rational Splines
- Spline Surfaces
- Subdivision Surfaces
- Implicit Functions
- Variational Modeling
- Point Based Representations
- Multi Resolution Representations
- Surface Parametrization

Overview

Modeling Techniques

Geometric Modeling

What do we want to do?

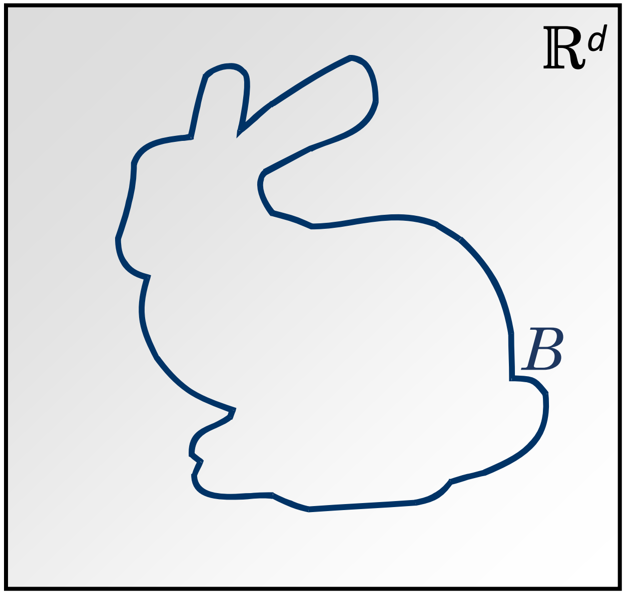


empty space
(typically $\mathbb{R}^3$)

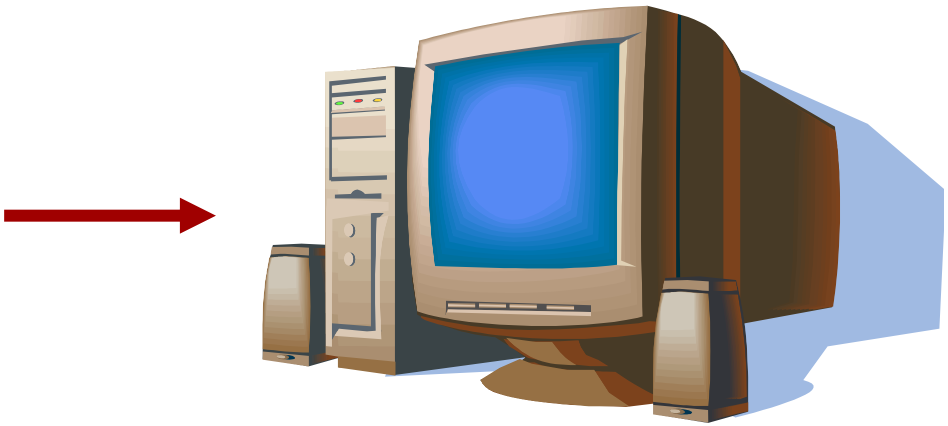
geometric object
$B \subseteq \mathbb{R}^3$

Fundamental Problem

The Problem:



infinite number of points



my computer: 8GB of memory

We need to encode a continuous model with a finite amount of information

Modeling Approaches

Two Basic Approaches

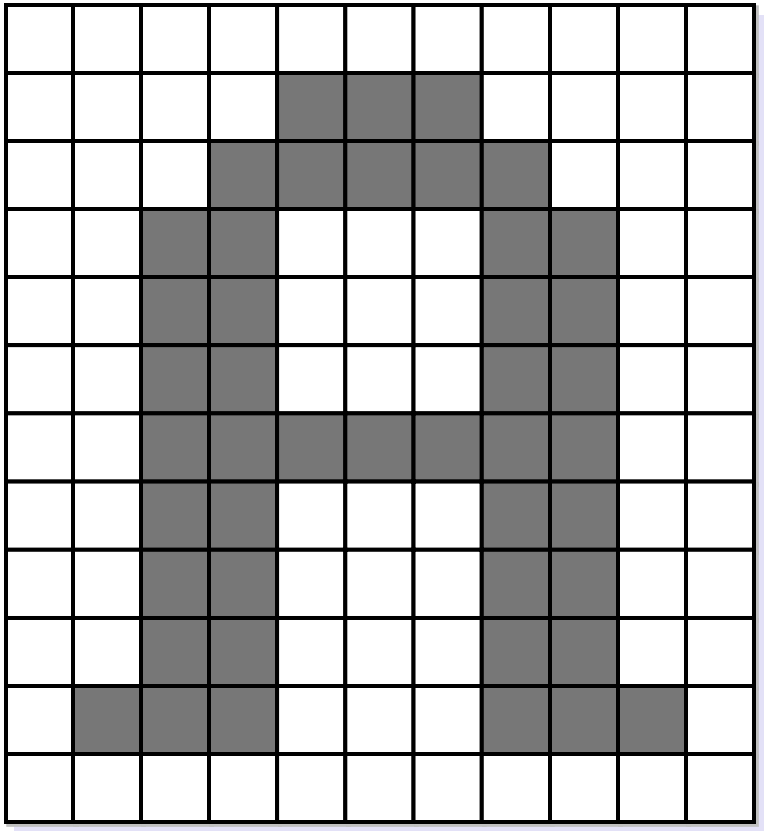
- Discrete representations
  - Fixed discrete bins
- "Continuous" representations
  - Mathematical description
  - Evaluate continuously

Discrete Representations

You know this...

- Fixed Grid of values:

  $(i_1, \ldots, i_{d_s}) \in \mathbb{Z}^{ds} \rightarrow (x_1, \ldots, x_{d_t}) \in \mathbb{R}^{dt}$
- Typical scenarios:
  - $d_s = 2$, $d_t = 3$: Bitmap images
  - $d_s = 3$, $d_t = 1$: Volume data (scalar fields)
  - $d_s = 2$, $d_t = 1$: Depth maps (range images)
- PDEs: "Finite Differences" models



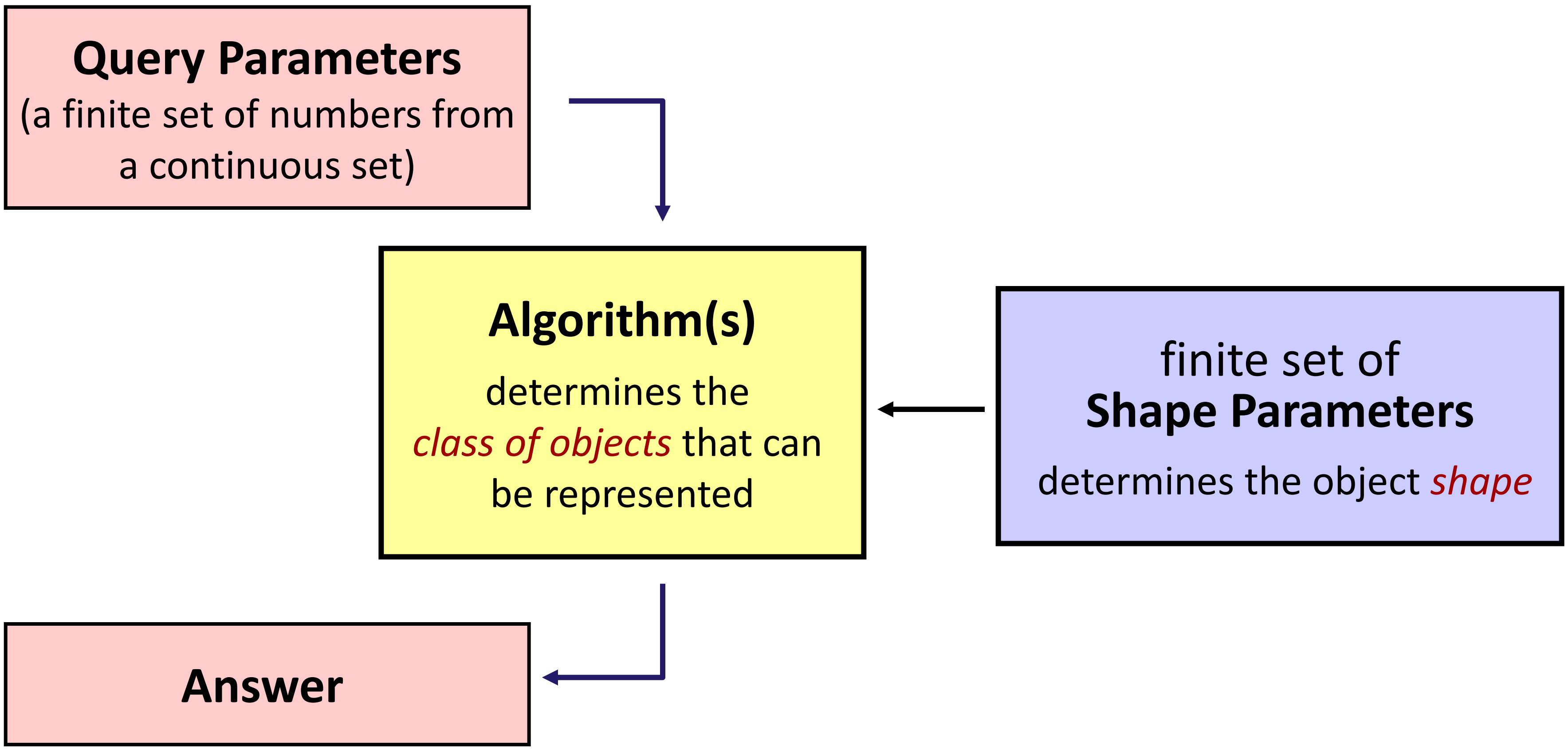
Modeling Approaches

Two Basic Approaches

- Discrete representations
  - Fixed discrete bins
- "Continuous" representations
  - Mathematical description
  - Evaluate continuously

Continuous Models

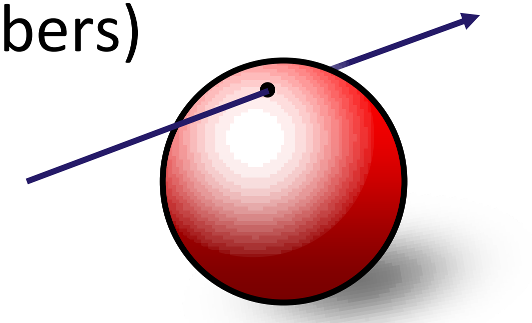
Basic Principle: Procedural Modeling

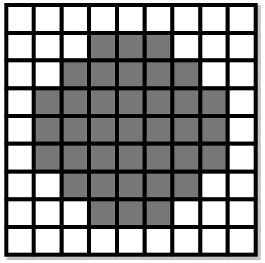


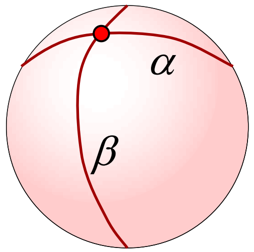
Example: Continuous Model

Example: Sphere

- Shape Parameters: center, radius (4 numbers)
- Algorithms:
  - **Ray Intersection** (e.g. for display)
    - Input: Ray (angle, position: 5 numbers)
    - Output: {true, false}
  - **Inside/outside test** (e.g. for rasterization)
    - Input: Position (3 numbers)
    - Output: {true, false}
  - **Parametrization** (e.g. for display)
    - Input: longitude, latitude (α, β)
    - Output: position (3 numbers)







So Many Questions...

Several algorithms for the same representation:

- *Parametrization* – compute surface points according to continuous parameters
- *(Signed) distance computation* – distance to surface of points in space, inside/outside test
- *Intersection* – with rays (rendering), other objects (collision detection)
- *Conversion* – into other representations.
- Many more...

In addition, we also need algorithms to construct and alter the models.

Continuous, Procedural Models

“Continuous” representations

- An algorithm describes the shape
- The shape is determined by a *finite number* of *continuous parameters*
- The shape can be queried with a *finite number* of *continuous parameters*
- More involved (have to ask for information)
- But potentially “infinite” resolution (continuous model)
- Structural model complexity still limited by algorithm and parameters

This lecture examines these representations and the corresponding algorithms

Classes of Models

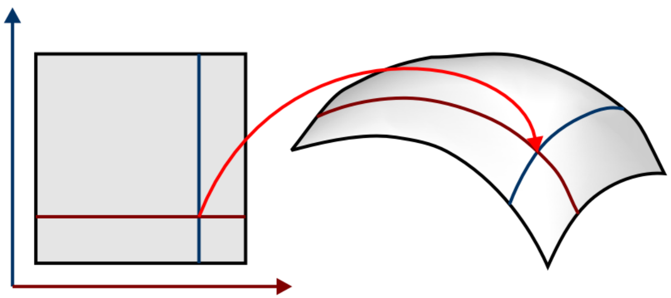
(Main) classes of models in this lecture:

- Primitive meshes
- Parametric models
- Implicit models
- Particle / point-based models

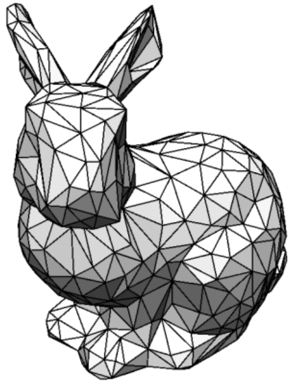
Remarks

- Most models are hybrid (combine several of these)
- Representations can be converted (may be approximate)
- Some questions are much easier to answer for certain representations

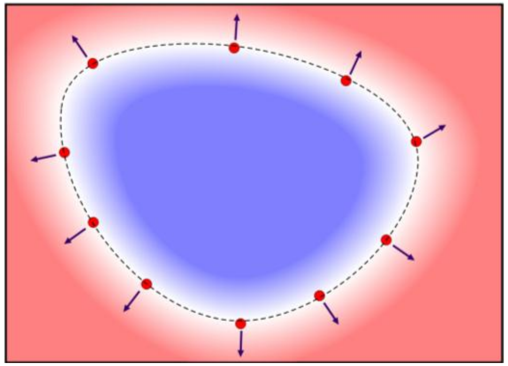
Modeling Zoo



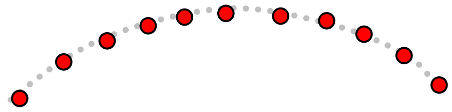
Parametric Models



Primitive Meshes

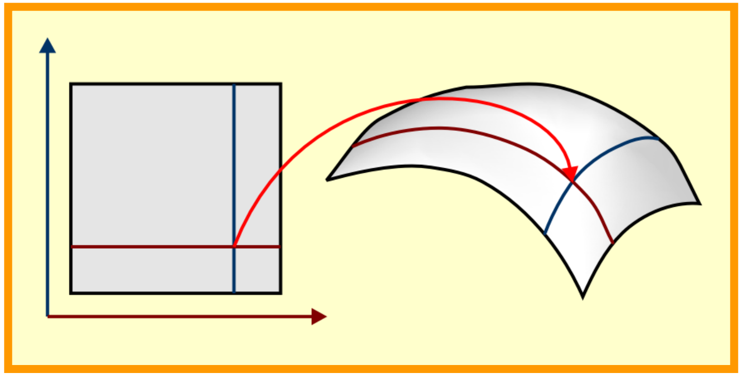


Implicit Models

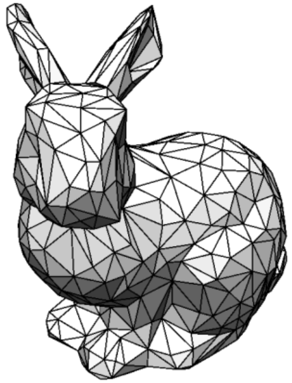


Particle Models

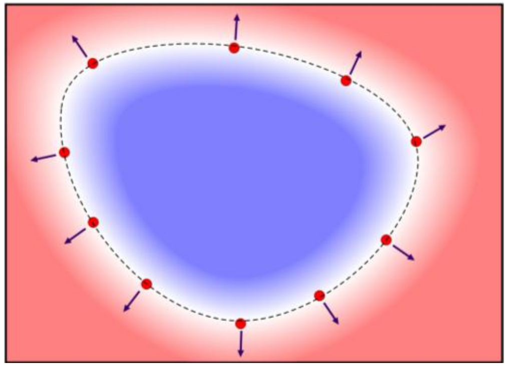
Modeling Zoo



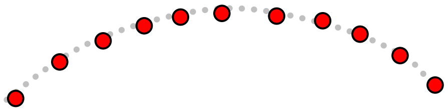
Parametric Models



Primitive Meshes

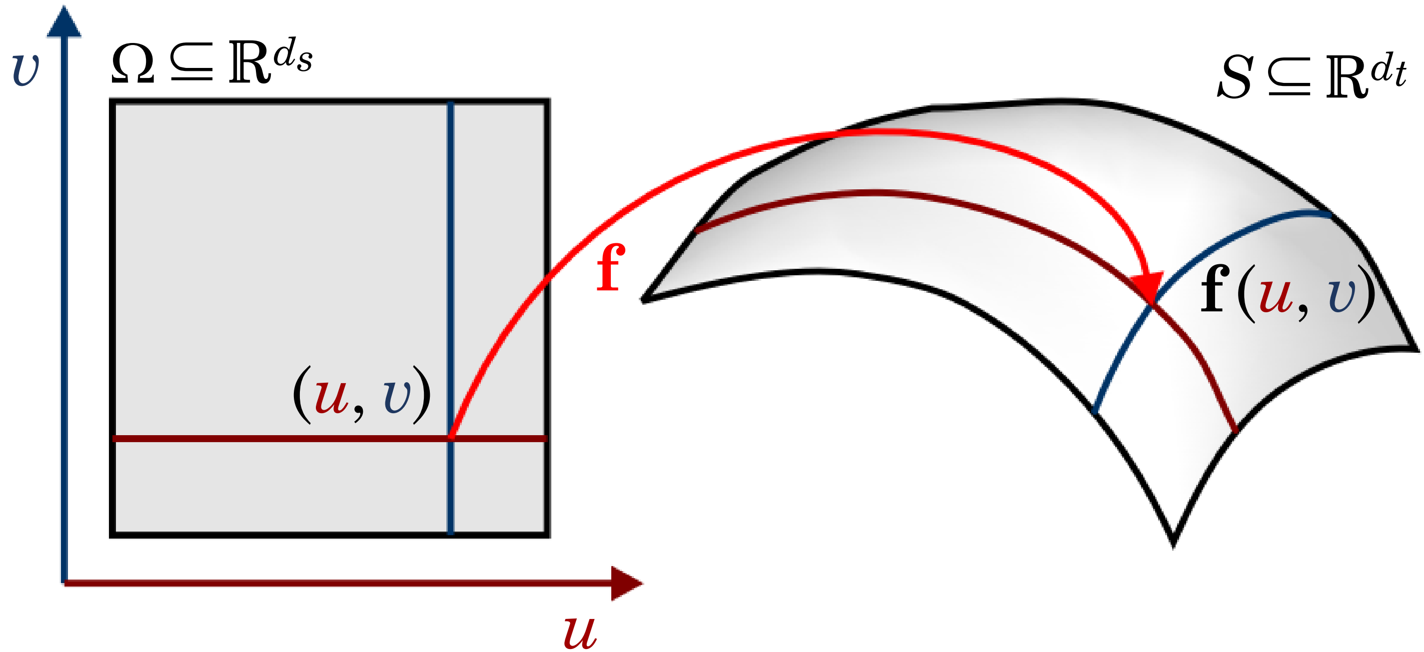


Implicit Models



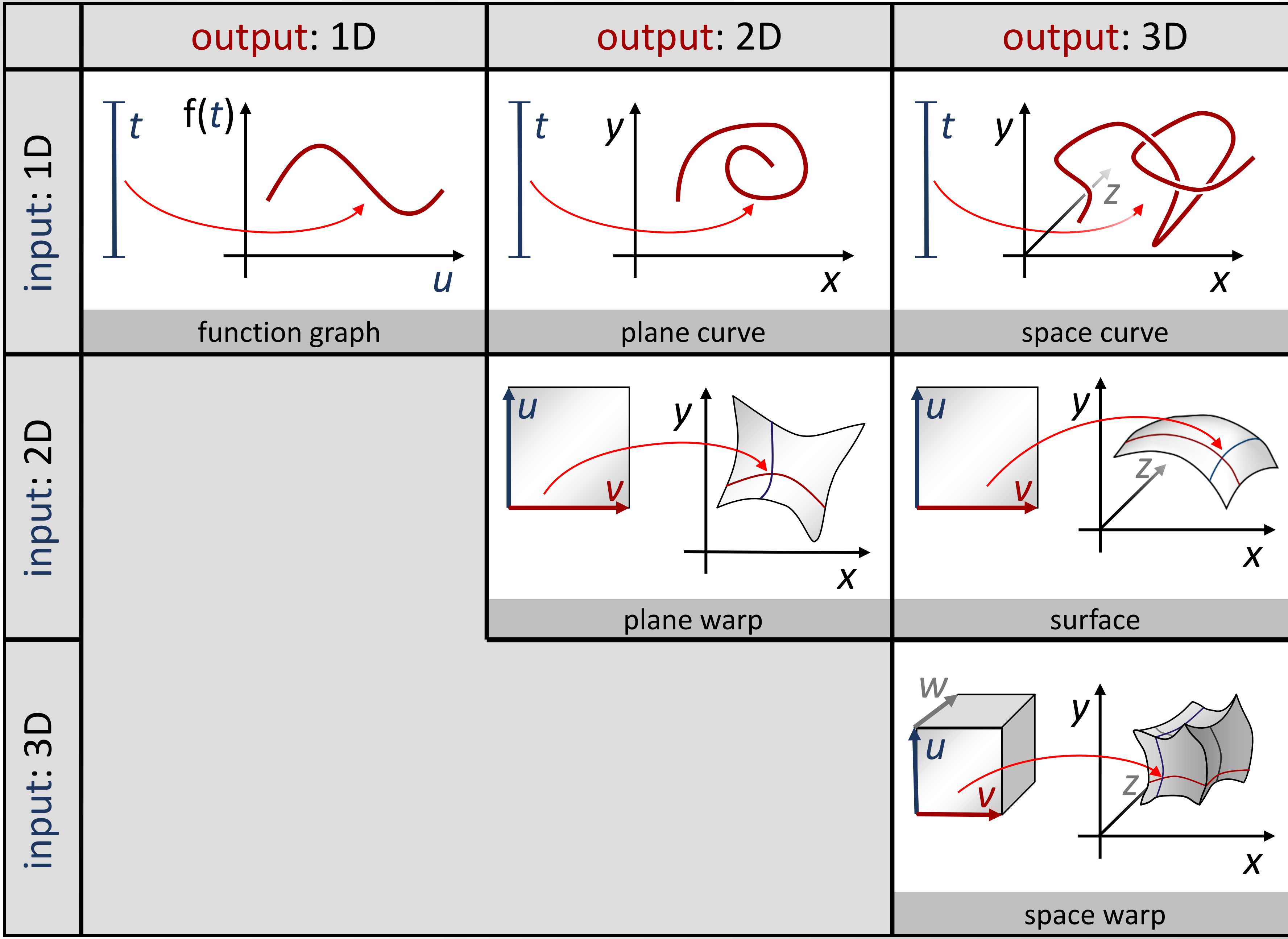
Particle Models

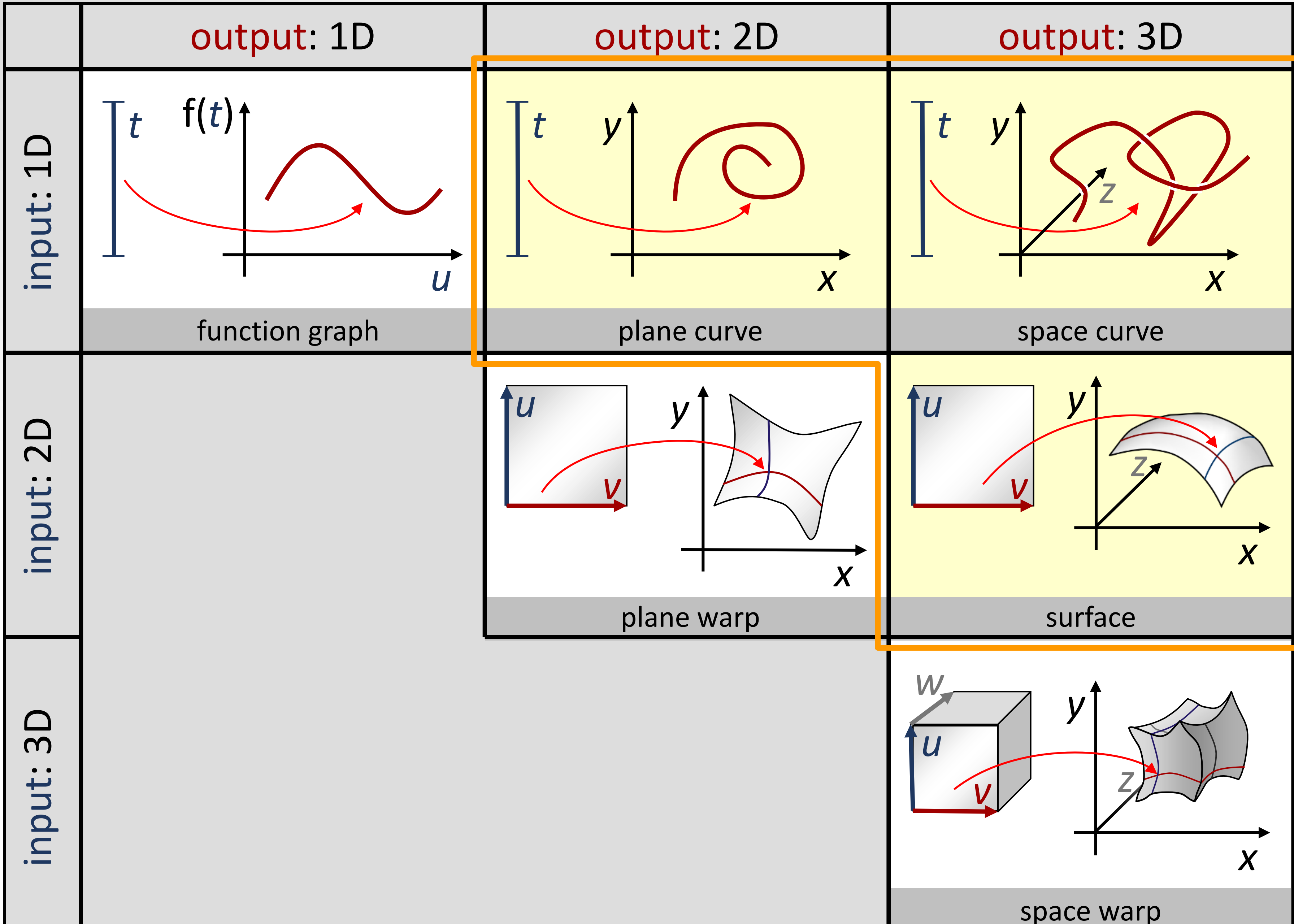
Parametric Models



Parametric Models

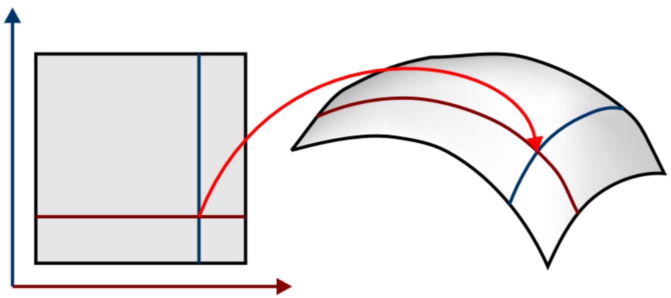
- Function **f** maps from parameter domain Ω to target space
- Evaluation of **f** gives one point on the model


output: 1D
output: 2D
output: 3D
input: 1D
input: 2D
input: 3D
t
f(t)
u
function graph
t
y
x
plane curve
t
y
z
x
space curve
u
v
y
x
plane warp
u
v
y
z
x
surface
w
u
v
y
z
x
space warp

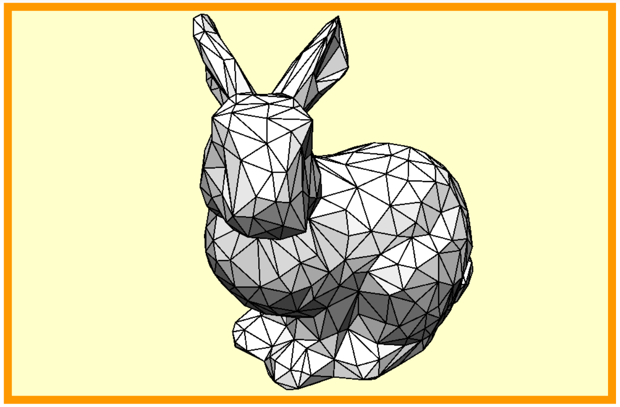


output: 1D
output: 2D
output: 3D
input: 1D
input: 2D
input: 3D
t
f(t)
u
y
x
z
v
w
function graph
plane curve
space curve
plane warp
surface
space warp

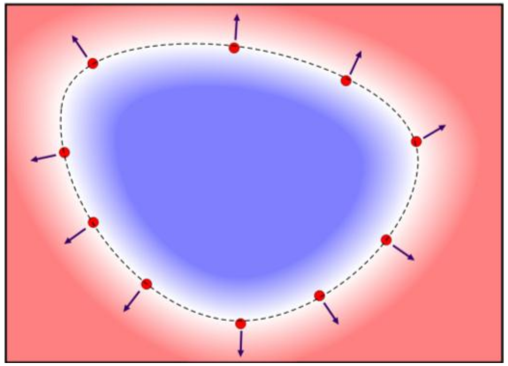
Modeling Zoo



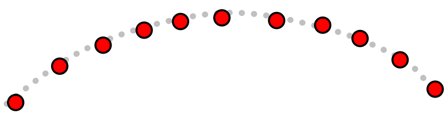
Parametric Models



Primitive Meshes



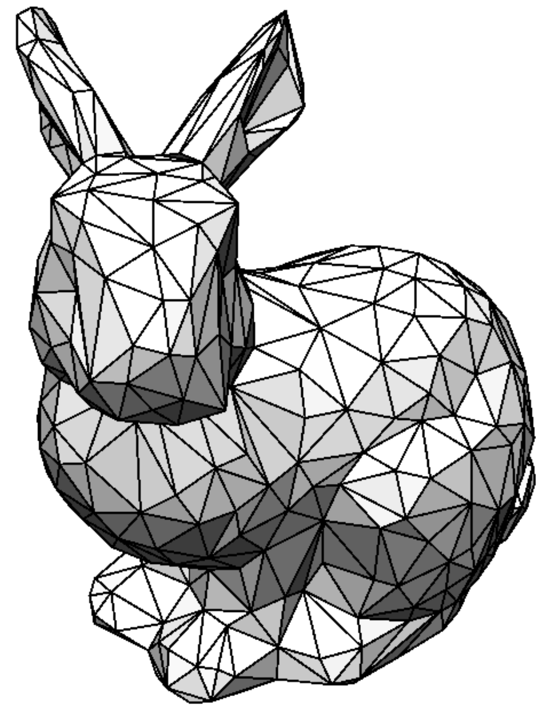
Implicit Models



Particle Models

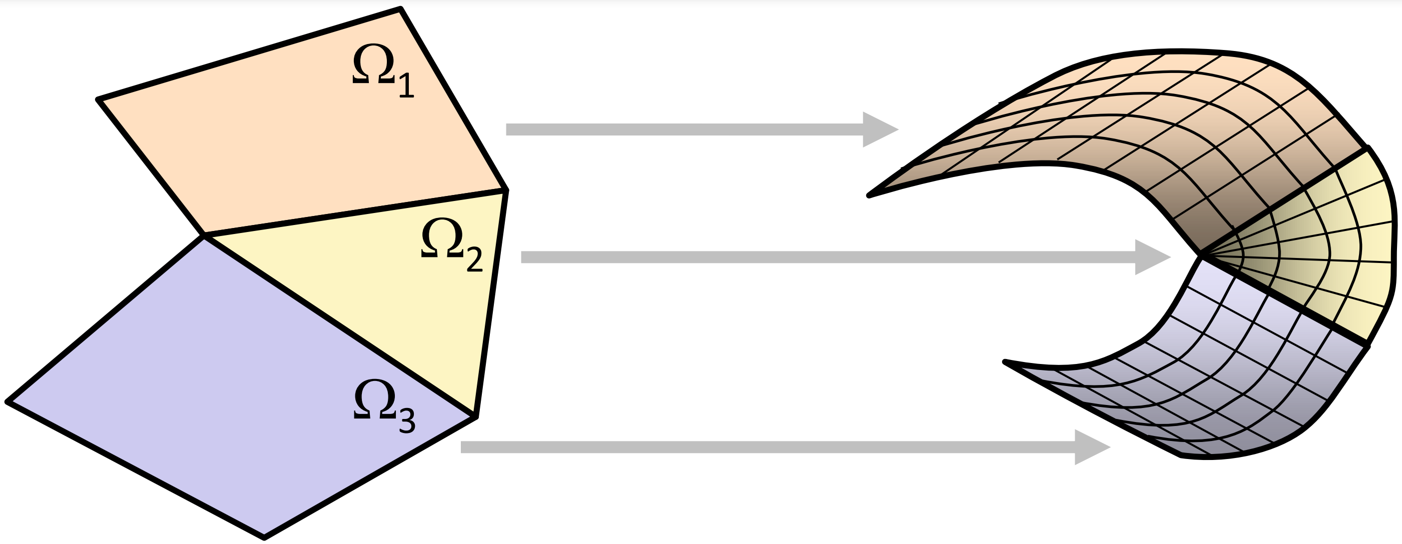
Primitive Meshes

Primitive Meshes



- Collection of geometric primitives
  - Triangles
  - Quadrilaterals
  - More general primitives (spline patches)
- Typically, the primitives are parametric surfaces
- Composite model:
  - Mesh encodes topology, rough shape
  - Primitive parameter encode local geometry
- Triangle meshes rule the world (“triangle soup”)

Primitive Meshes



Complex Topology for Parametric Models

- Mesh of parameter domains attached in a mesh
- Domain can have complex shape ("trimmed patches")
- Separate mapping function *f* for each part
  (typically of the same class)

Meshes are Great

Advantages of mesh-based modeling:

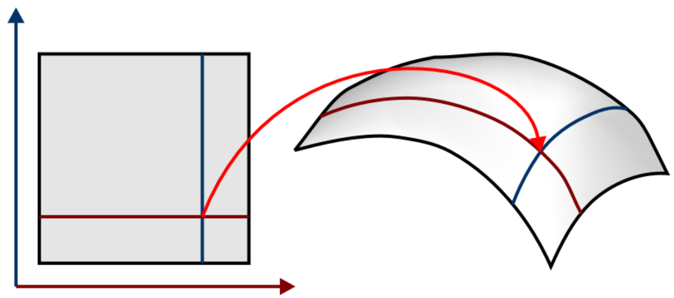
- Compact representation (usually)
- Can represent arbitrary topology
- Using the right parametric surfaces as parts, many important geometric objects can be represented exactly (e.g. NURBS: circles, cylinders, spheres $\rightarrow$ CAD/CAM)

Meshes are not so great

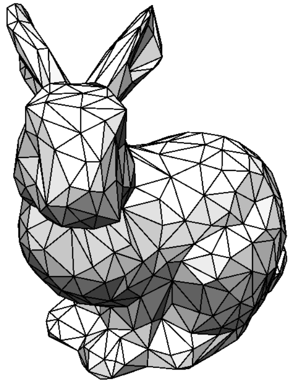
Problem with Meshes:

- Need to specify a mesh first, then edit geometry
- Problems for larger changes
  - Mesh structure and shape need to be adjusted
  - Mesh encodes object topology
    $\Rightarrow$ Changing object topology is painful
- Difficult to use for many applications (such as surface reconstruction)
  - Rule of thumb: If the topology or the coarse scale shape changes drastically and frequently during computations, meshes are hard to use
  - Drastic example: Fluid simulation (surface of splashing water)

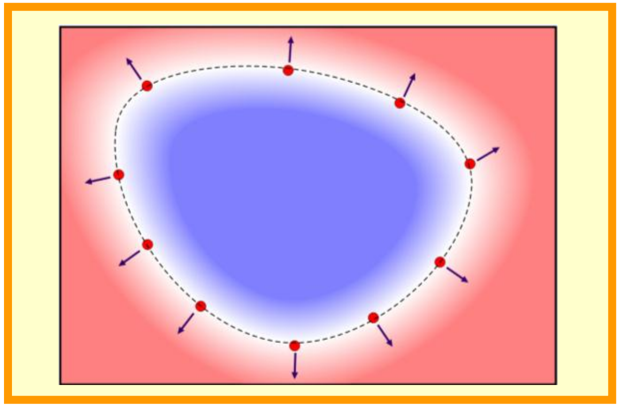
Modeling Zoo



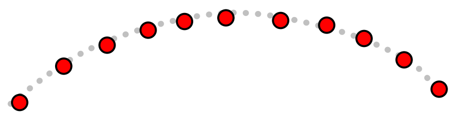
Parametric Models



Primitive Meshes



Implicit Models



Particle Models

Implicit Modeling

General Formulation:

- Curve / Surface $S = \{\mathbf{x} \mid f(\mathbf{x}) = 0\}$
- $\mathbf{x} \in \mathbb{R}^d$ (d = 2,3), $f(\mathbf{x}) \in \mathbb{R}$
- S is (usually) a d-1 dimensional object

This means...:

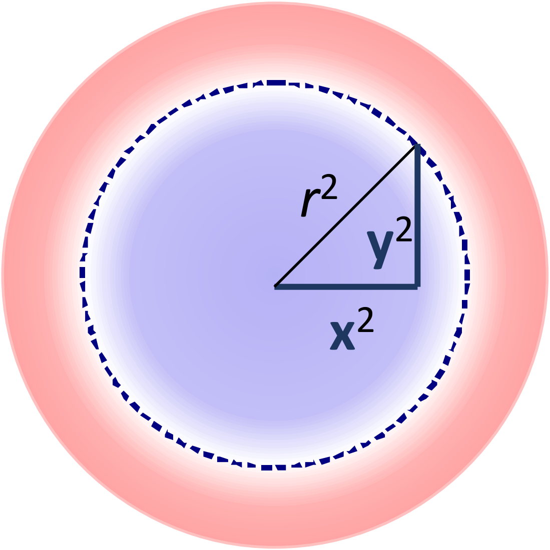
- The surface is obtained implicitly as the set of points for which some given function vanishes ($f(\mathbf{x}) = 0$)
- Alternative notation: $S = f^{-1}(0)$ ("inverse" yields a set)

Implicit Modeling

Example:

- Circle: $\mathbf{x}^2 + \mathbf{y}^2 = r^2$

  $\Leftrightarrow f_r(\mathbf{x},\mathbf{y}) = \mathbf{x}^2 + \mathbf{y}^2 - r^2 = 0$
- Sphere: $\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 = r^2$



Special Case:

- Signed distance field
- Function value is signed distance to surface

$$f(\mathbf{x},\mathbf{y}) = sign(\mathbf{x}^2 + \mathbf{y}^2 - \mathbf{r}^2)\sqrt{|\,\mathbf{x}^2 + \mathbf{y}^2 - \mathbf{r}^2\,|}$$

- Negative means inside, positive means outside

Implicit Modeling

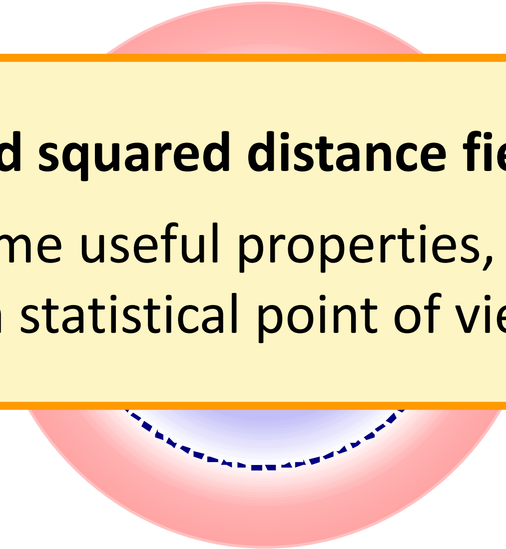
Example:

- Circle: $\mathbf{x}^2 + \mathbf{y}^2 = r^2$

  $\Leftrightarrow f_r(\mathbf{x},\mathbf{y}) = \mathbf{x}^2 + \mathbf{y}^2 - r^2 = 0$
- Sphere: $\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 = r^2$

“Signed squared distance field”

(has some useful properties, e.g. from a statistical point of view)



Special Case:

- Signed distance field
- Function value is signed distance to surface

$$f(\mathbf{x},\mathbf{y}) = sign(\mathbf{x}^2 + \mathbf{y}^2 - \mathbf{r}^2)\sqrt{|\mathbf{x}^2 + \mathbf{y}^2 - \mathbf{r}^2|}$$

- Negative means inside, positive means outside

Implicit Modeling: Pros & Cons

Advantages:

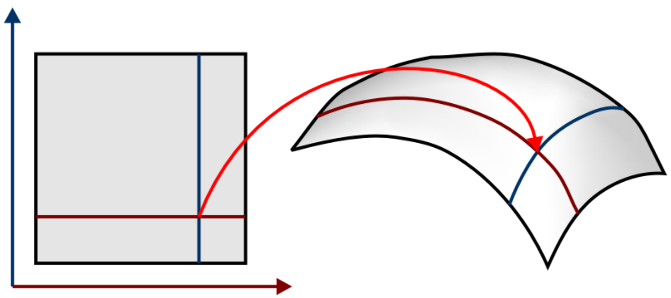
- More general than parametric techniques
- Topology can be changed easily
  (depends on how f is specified, though)
- Implicit representations are the standard technique for simulations with *free boundaries*.
  Also known as "*level-set methods*".
- Typical example: Fluid simulation (evolving water-air interface)
- Geometric modeling: Surface reconstruction, "blobby surfaces"

Implicit Modeling: Pros & Cons

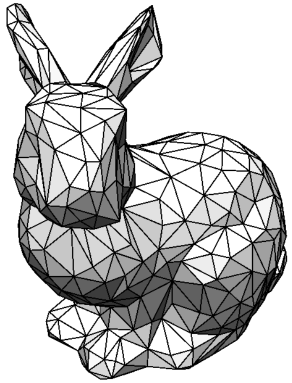
Disadvantages:

- Need to solve inversion problem $S = f^{-1}(0)$
- Algorithms for display, conversion etc. tend to be more difficult and more expensive (inside/outside test is easy though for signed distance fields)
- Representing objects takes more memory
(we will discuss standard representations later)

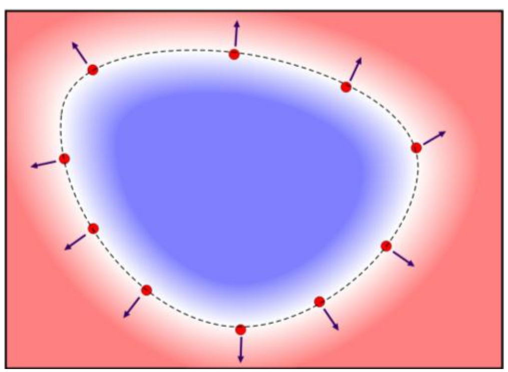
Modeling Zoo



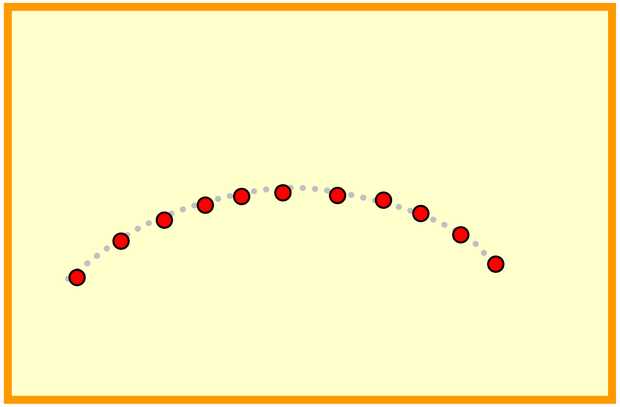
Parametric Models



Primitive Meshes



Implicit Models

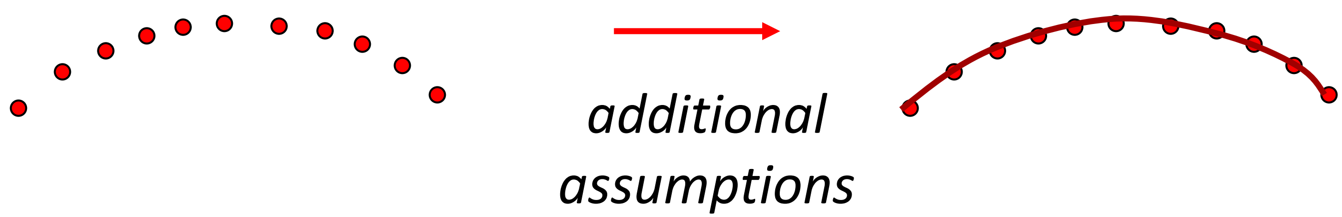


Particle Models

Particle Representations

Particle / Point-based Representations

- Geometry is represented as a set of points / particles
- The particles form a (typically irregular) *sample* of the geometric object
- Need additional information to deal with "the empty space around the particles"



Particle Representations

Helpful Information

- Each particle may carries a set of attributes
  - Must have: Its position
  - Additional geometric information:
    Particle density (sample spacing), surface normals
  - Additionally: Color, physical quantities (mass, pressure, temperature), ...
- This information can be used to improve the *reconstruction* of the geometric object described by the particles

The Wrath of Khan

Why Star Trek is at fault...

- Particle methods first used in computer graphics to represent fuzzy phenomena (fire, clouds, smoke)
- “*Particle Systems—a Technique for Modeling a Class of Fuzzy Objects*” [Reeves 1984]
- Probably most well-known example: Genesis sequence

Genesis Sequence [Reeves 1983]

Non-Fire Objects

Particle Traces for Modeling Plants

(also from [Reeves 1983])

Geometric Modeling

How became the geometric modeling crowd interested in this?

3D Scanners

- 3D scanning devices yield point clouds
  (often: measure distance to points in space, one at a time)
- Then you have to deal with the problem anyway
- Need algorithms to directly work on “point clouds”
  (this is the geometry name for particle system)

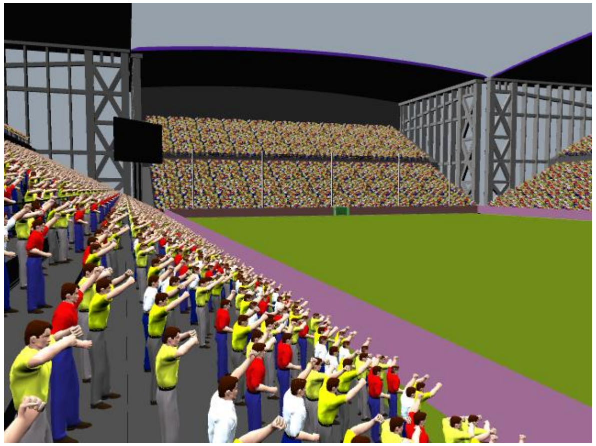
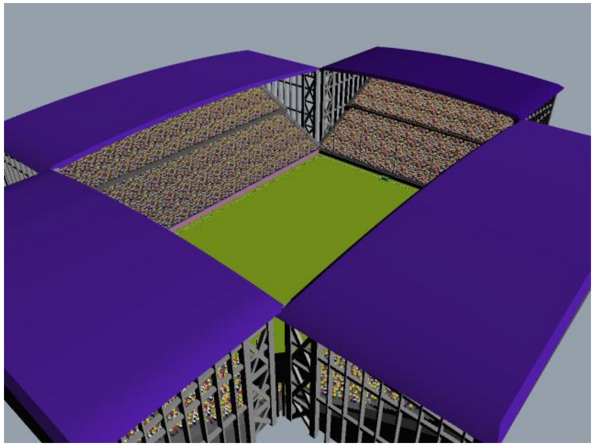
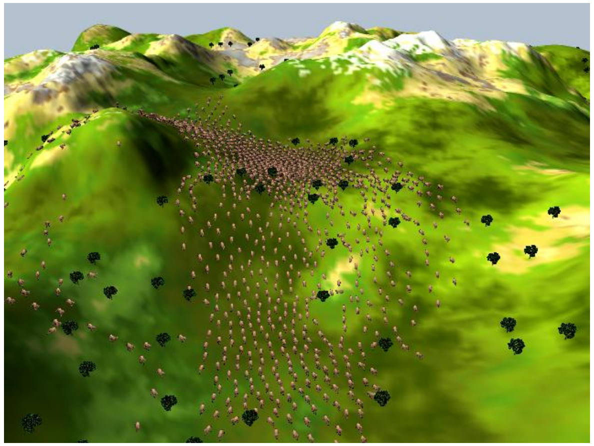
Geometric Modeling

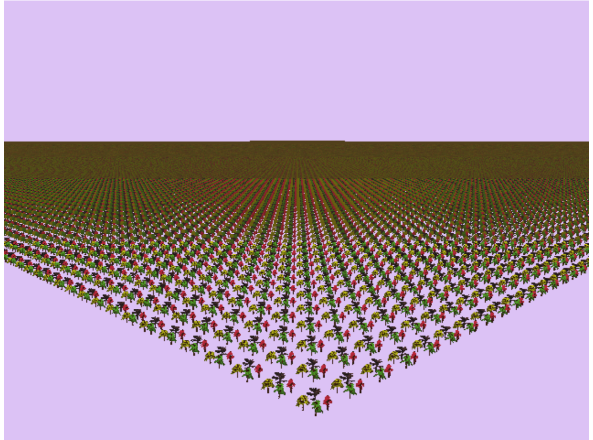
How became the geometric modeling crowd interested in this?

Other Reasons:

- Similar advantages as implicit techniques
- Topology does not matter (for the good and for the bad)
  - Topology is easy to change
  - Multi-scale representations are easy to do
    (more details on multi-resolution techniques later)
- Often easier to use than implicit or parametric techniques

Multi-Scale Geometry w/Points




Summary

Summary

- Lots of different representations
- No silver bullet
- In theory, everything always works, but might be just too complicated/expensive
- Best choice depends on the application
- We will look on all of this...
  - Focus on parametric techniques though
  - Most common approach