Maximum Value Precedence Algorithm

- greedy heuristics can produce poor results
- IKKBZ only support acyclic queries and ASI cost functions
- Maximum Value Precedence (MVP) [4] algorithm is a polynomial time heuristic with good results
- considers join ordering a graph theoretic problem

Directed Join Graph

Given a conjunctive query with predicates P.

- for all join predicates $p \in P$, we denote by $\mathcal{R}(p)$ the relations whose attributes are mentioned in p.
- the *directed join graph* of the query is a triple $G = (V, E_p, E_v)$, where V is the set of predicates and E_p and E_v are sets of directed edges defined as follows
- for any nodes $u, v \in V$, if $\mathcal{R}(u) \cap \mathcal{R}(v) \neq \emptyset$ then $(u, v) \in E_p$ and $(v, u) \in E_p$
- if $\mathcal{R}(u) \cap \mathcal{R}(v) = \emptyset$ then $(u, v) \in E_v$ and $(v, u) \in E_v$
- edges in E_p are called *physical edges*, those in E_v *virtual edges*

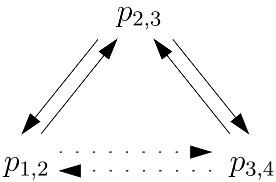
Note: all nodes u, v there is an edge (u, v) that is either physical or virtual. Hence, G is a clique.

Examples: Spanning Tree and Join Tree

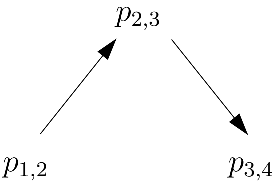
- every spanning tree in the directed join graph leads to a join tree

$$R_1 - R_2 - R_3 - R_4$$

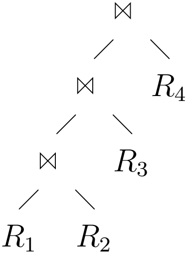
query graph



directed join graph

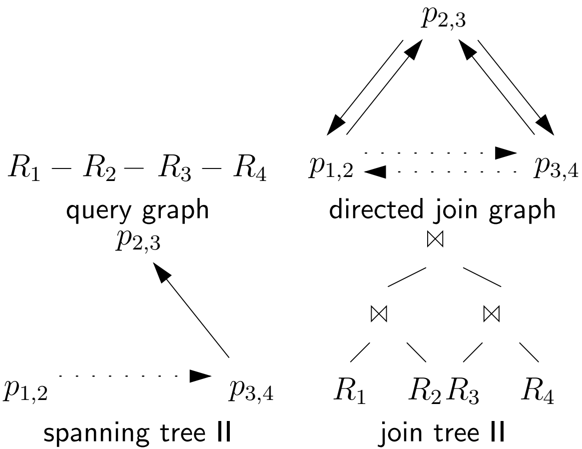


spanning tree I



join tree I

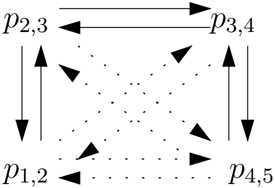
Examples: Spanning Tree and Join Tree (2)



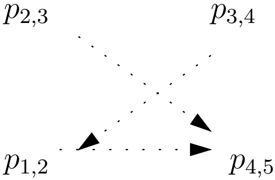
Examples: Spanning Tree and Join Tree (3)

$$R_1 - R_2 - R_3 - R_4 - R_5$$

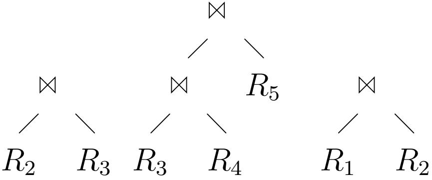
query graph



directed join graph



spanning tree III



join tree III (?)

- spanning tree does not correspond to a (effective) join tree!

Effective Spanning Trees

It can be shown that a spanning tree $T = (V, E)$ is *effective*, it is satisfies the following conditions:

1. T is a binary tree
2. for all inner nodes v and nodes u with $(u, v) \in E$:
   $\mathcal{R}(T(u))) \cap \mathcal{R}(v) \neq \emptyset$
3. for all nodes v, u_1, u_2 with $u_1 \neq u_2, (u_1, v) \in E$ and $(u_2, v) \in E$ one of the following conditions holds:
   3.1 $((\mathcal{R}(T(u_1)) \cap \mathcal{R}(v)) \cap (\mathcal{R}(T(u_2)) \cap \mathcal{R}(v))) = \emptyset$ or
   3.2 $(\mathcal{R}(T(u_1)) = \mathcal{R}(v)) \vee (\mathcal{R}(T(u_2)) = \mathcal{R}(v))$

We denote by $T(v)$ the partial tree rooted at v.

Adding Weights to the Edges

For two nodes $v, u \in V$ we define $u \sqcap v = \mathcal{R}(u) \cap \mathcal{R}(v)$

- for simplicity, we assume that every predicate involves exactly two relations
- then for all $u, v \in V$, $a \sqcap v$ contains a single relation (or none)

Let $v \in V$ be a node with $\mathcal{R}(v) = \{R_i, R_j\}$

- we abbreviate $R_i \bowtie_v R_j$ by $\bowtie_v$

Using these notations, we can attach weights to the edges to define the *weighted directed join graph*.

Adding Weights to the Edges (2)

Let $G = (V, E_p, E_v)$ be a directed join graph for a conjunctive query with join predicates P. The *weighted directed join graph* is derived from G by attaching a weight to each edge as follows:

- Let $(u, v) \in E_p$ be a physical edge. The weight $w_{u,v}$ of (u, v) is defined as

$$w_{u,v} = \frac{|\bowtie_u|}{|u \sqcap v|}$$

- For virtual edges $(u, v) \in E_v$, we define

$$w_{u,v} = 1$$

Note that $w_{u,v}$ is not symmetric.

Remark on Edge Weights

The weights of physical edges are equal to the s_i used in the IKKBZ-Algorithm.
Assume $\mathcal{R}(u) = \{R_1, R_2\}, \mathcal{R}(v) = \{R_2, R_3\}$. Then

$$\begin{aligned} w_{u,v} &= \frac{|\bowtie_u|}{|u \sqcap v|} \\ &= \frac{|R_1 \bowtie R_2|}{|R_2|} \\ &= \frac{f_{1,2}|R_1||R_2|}{|R_2|} \\ &= f_{1,2}|R_1| \end{aligned}$$

Hence, if the join $R_1 \bowtie_u R_2$ is executed before the join $R_2 \bowtie_v R_3$, the input size to the latter join changes by a factor of $w_{u,v}$

Adding Weights to the Nodes

- the weight of a node reflects the change in cardinality to be expected when certain other joins have been executed before
- it depends on a (partial) spanning tree S

Given S, we denote by $\bowtie^S_{p_{i,j}}$ the result of the join $\bowtie_{p_{i,j}}$ if all joins preceding $p_{i,j}$ in S have been executed. Then the weight attached to node $p_{i,j}$ is defined as

$$w(p_{i,j}, S) = \frac{|\bowtie^S_{p_{i,j}}|}{|R_i \bowtie_{p_{i,j}} R_j|}$$

For empty sequences we define $w(p_{i,j}, \epsilon) = |R_i \bowtie_{p_{i,j}} R_j|$.
Similarly, we define the cost of a node $p_{i,j}$ depending on other joins preceding it in some given spanning tree S. We denote this by $C(p_{i,j}, S)$.

- the actual cost function can be chosen arbitrarily
- if we have several join implementations: take the minimum

Algorithm Overview

The algorithm builds an effective spanning tree in two phases:

1. it takes those edges with a weight < 1
2. it adds the remaining edges

keeping track of effectiveness during the process.

- rational: weight < 1 is good
- decreases the work for later operators
- should be done early
- increasing intermediate results as late as possible

MVP Algorithm

MVP(G)
Input: a weighted directed join graph $G = (V, E_p, E_v)$
Output: an effective spanning tree
Q_1 = a priority queue for nodes, smallest w first
Q_2 = a priority queue for nodes, largest w first
insert all nodes in V to Q_1
$G' = (V', E')$ with $V' = V$ and $E' = E_p$ // working graph
$S = (V_S, E_s)$ with $V_s = V$ and $E_s = \emptyset$ // result
MVP-Phase1(G, G', S, Q_1, Q_2)
MVP-Phase2(G, G', S, Q_1, Q_2)
return S

MVP Algorithm (2)

MVP-Phase1(G, G', S, Q_1, Q_2)
Input: state from MVP
Output: modifies the state
while $|Q_1| > 0 \wedge |E_s| < |V| - 1$ {
  $v =$ head of Q_1
  $U = \{u | (u, v) \in E' \wedge w_{u,v} < 1 \wedge (V, E_S \cup \{(u, v)\})$ is acyclic and effective}
  if $U = \emptyset$ {
    $Q_1 = Q_1 \setminus \{v\}$
    $Q_2 = Q_2 \cup \{v\}$
  } **else** {
    $u = \arg\max_{u \in U} C(\bowtie_v, S) - C(\bowtie_v, (V, E_S \cup \{(u, v)\}))$
    MVPUpdate($G, G', S, (u, v)$)
    recompute w for v and its ancestors
  }
}

MVP Algorithm (3)

MVP-Phase2(G, G', S, Q_1, Q_2)
Input: state from MVP
Output: modifies the state
while $|Q_2| > 0 \land |E_s| < |V| - 1$ {
  $v =$ head of Q_2
  $U = \{(x,y) | (x,y) \in E' \land (x = v \lor y = v) \land (V, E_S \cup \{(x,y)\})$ is acyclic and effective$\}$
  $(x,y) = \arg\min_{(x,y) \in U} C(\Join_v, (V, E_S \cup \{(x,y)\})) - C(\Join_v, S)$
  MVPUpdate($G, G', S, (x,y)$)
  recompute w for y and its ancestors
}

MVP Algorithm (4)

MVPUpdate($G, G', S, (u, v)$)
Input: state from MVP, an edge to be added to S
Output: modifies the state
$E_S = E_S \cup \{(u, v)\}$
$E' = E' \setminus \{(u, v), (v, u)\}$
$E' = E' \setminus \{(u, w)|(u, w) \in E'\}$
$E' = E' \cup \{(v, w)|(u, w) \in E_p, (v, w) \in E_v\}$
if v has two incoming edges in S {
  $E' = E' \setminus \{(w, v)|(w, v) \in E'\}$
}
if v has one outflowing edge in S {
  $E' = E' \setminus \{(v, w)|(v, w) \in E'\}$
}

- checks that S is a tree (one parent, at most two children)
- detects transitive physical edges

Dynamic Programming

Basic premise:

- optimality principle
- avoid duplicate work

A very generic class of approaches:

- all cost functions (as long as optimality principle holds)
- left-deep/bushy, with/without cross products
- finds the optimal solution

Concrete algorithms can be more specialized of course.

Optimality Principle

Consider the two joins trees

$$(((R_1 \bowtie R_2) \bowtie R_3) \bowtie R_4) \bowtie R_5$$

and

$$(((R_3 \bowtie R_1) \bowtie R_2) \bowtie R_4) \bowtie R_5$$

- if we know that $((R_1 \bowtie R_2) \bowtie R_3)$ is cheaper than $((R_3 \bowtie R_1) \bowtie R_2)$, we know that the first join is cheaper than the second join
- hence, we could avoid generating the second alternative and still won't miss the optimal join tree

Optimality Principle (2)

More formally, the optimality for join ordering:

Let T be an optimal join tree for relations $R_1, \ldots, R_n$. Then, every subtree S of T must be an optimal join tree for the relations contained in it.

- optimal substructure: the optimal solution for a problem can be constructed from optimal solutions to its subproblems
- not true with physical properties (but can be fixed)

Overview Dynamic Programming Strategy

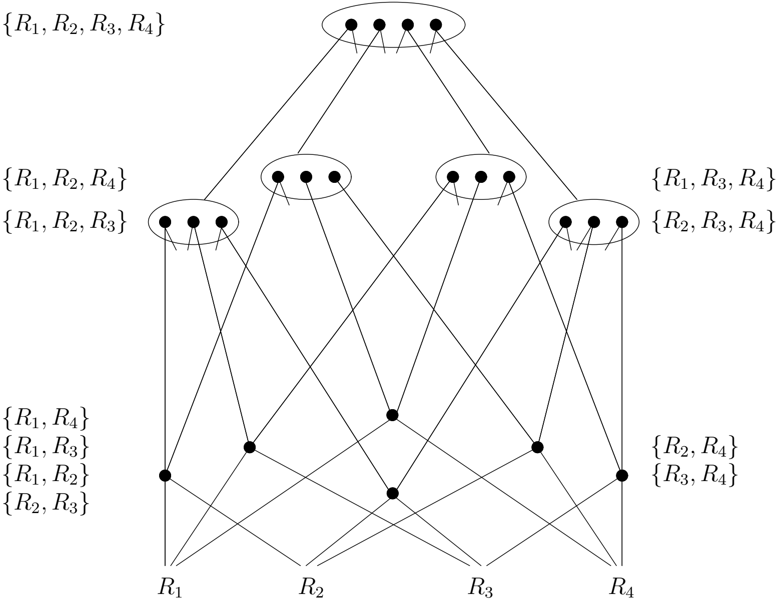
- generate optimal join trees bottom up
- start from optimal join trees of size one (relations)
- build larger join trees by (re-)using those of smaller sizes

To keep the algorithms concise, we use a subroutine *CreateJoinTree* that joins two trees.

Creating Join Trees

CreateJoinTree(T_1, T_2)
Input: two (optimal) join trees T_1, T_2
for linear trees: assume that T_2 is a single relation
Output: an (optimal) join tree for $T_1 \bowtie T_2$
$B = \emptyset$
for each *impl* $\in$ { applicable join implementations } {
  if $\neg$right-deep only {
    $B = B \cup \{T_1 \bowtie^{impl} T_2\}$
  }
  if $\neg$left-deep only {
    $B = B \cup \{T_2 \bowtie^{impl} T_1\}$
  }
}
return $\arg\min_{T \in B} C(T)$

Search Space with Sharing under Optimality Principle



Generating Linear Trees

- a (left-deep) linear tree T with $|T| > 1$ has the form $T' \bowtie R_i$, with $|T| = |T'| + 1$
- if T is optimal, T' must be optimal too
- basic strategy: find the optimal T by joining all optimal T' with $T \setminus T'$

enumeration order varies between algorithms

Generating Linear Trees (2)

DPsizeLinear(R)
Input: a set of relations $R = \{R_1, \ldots, R_n\}$ to be joined
Output: an optimal left-deep (right-deep, zig-zag) join tree
B = an empty DP table $2^R \rightarrow$ join tree
for each $R_i \in R$
  $B[\{R_i\}] = R_i$
for each $1 < s \leq n$ **ascending** {
  for each $S \subset R, R_i \in R : |S| = s - 1 \wedge R_i \notin S$ {
    if $\neg$cross products $\wedge \neg S$ connected to R_i **continue**
    $p_1 = B[S], p_2 = B[\{R_i\}]$
    if $p_1 = \epsilon$ **continue**
    P = CreateJoinTree(p_1, p_2);
    if $B[S \cup \{R_i\}] = \epsilon \vee C(B[S \cup \{R_i\}]) > C(P)$
      $B[S \cup \{R_i\}] = P$
  }
}
return $B[\{R_1, \ldots, R_n\}]$

Order in which Subtrees are generated

The ordering in which subtrees are generated does not matter as long as the following condition is not violated:

> *Let S be a subset of $\{R_1, \ldots, R_n\}$. Then, before a join tree for S can be generated, the join trees for all relevant subsets of S must already be available.*

- *relevant* means that they are valid subproblems by the algorithm
- usually this means connected (no cross products)

Generation in Integer Order

000	$\{\}$
001	$\{R_1\}$
010	$\{R_2\}$
011	$\{R_1, R_2\}$
100	$\{R_3\}$
101	$\{R_1, R_3\}$
110	$\{R_2, R_3\}$
111	$\{R_1, R_2, R_3\}$

- can be done very efficiently
- set representation is just a number

Generating Linear Trees (3)

DPsubLinear(R)
Input: a set of relations $R = \{R_1, \ldots, R_n\}$ to be joined
Output: an optimal left-deep (right-deep, zig-zag) join tree
B = an empty DP table $2^R \rightarrow$ join tree
for each $R_i \in R$
  $B[\{R_i\}] = R_i$
for each $1 < i \leq 2^n - 1$ **ascending** $\{$
  $S = \{R_j \in R | (\lfloor i/2^{j-1} \rfloor \, mod \, 2) = 1\}$
  for each $R_j \in S$ $\{$
    if $\neg$cross products $\wedge \neg S \setminus \{R_j\}$ connected to R_j **continue**
    $p_1 = B[S \setminus \{R_j\}], p_2 = B[\{R_j\}]$
    if $p_1 = \epsilon$ **continue**
    P = CreateJoinTree(p_1, p_2);
    if $B[S] = \epsilon \vee C(B[S]) > C(P)$ $B[S] = P$
  $\}$
$\}$
return $B[\{R_1, \ldots, R_n\}]$

Generating Bushy Trees

- a bushy tree T with $|T| > 1$ has the form $T_1 \bowtie T_2$, with $|T| = |T_1| + |T_2|$
- if T is optimal, both T_1 and T_2 must be optimal too
- basic strategy: find the optimal T by joining all pairs of optimal T_1 and T_2

Generating Bushy Trees (2)

DPsize(R)
Input: a set of relations $R = \{R_1, \ldots, R_n\}$ to be joined
Output: an optimal bushy join tree
B = an empty DP table $2^R \rightarrow$ join tree
for each $R_i \in R$
  $B[\{R_i\}] = R_i$
for each $1 < s \leq n$ **ascending** {
  for each $S_1, S_2 \subset R : |S_1| + |S_2| = s$ {
    if ($\neg$cross products $\wedge \neg S_1$ connected to S_2) $\vee$ ($S_1 \cap S_2 \neq \emptyset$) **continue**
    $p_1 = B[S_1], p_2 = B[S_2]$
    if $p_1 = \epsilon \vee p_2 = \epsilon$ **continue**
    P = CreateJoinTree(p_1, p_2);
    if $B[S_1 \cup S_2] = \epsilon \vee C(B[S_1 \cup S_2]) > C(P)$
      $B[S_1 \cup S_2] = P$
  }
}
return $B[\{R_1, \ldots, R_n\}]$

Generating Bushy Trees (3)

DPsub(R)
Input: a set of relations $R = \{R_1, \ldots, R_n\}$ to be joined
Output: an optimal bushy join tree
B = an empty DP table $2^R \rightarrow$ join tree
for each $R_i \in R$
  $B[\{R_i\}] = R_i$
for each $1 < i \leq 2^n - 1$ **ascending** {
  $S = \{R_j \in R | (\lfloor i/2^{j-1} \rfloor \, mod \, 2) = 1\}$
  for each $S_1 \subset S, S_2 = S \setminus S_1$ {
    if $\neg$cross products $\wedge \neg S_1$ connected to S_2 **continue**
    $p_1 = B[S_1], p_2 = B[S_2]$
    if $p_1 = \epsilon \vee p_2 = \epsilon$ **continue**
    P = CreateJoinTree(p_1, p_2);
    if $B[S] = \epsilon \vee C(B[S]) > C(P)$ $B[S] = P$
  }
}
return $B[\{R_1, \ldots, R_n\}]$

Efficient Subset Generation

If we use integers as set representation, we can enumerate all subsets of S as follows:

$S_1 = S\&(-S)$
do $\{$
  $S_2 = S - S_1$
  // Do something with S_1 and S_2
  $S_1 = S\&(S_1 - S)$
$\}$ **while** $(S_1 ! = S)$

- enumerates all subsets except $\emptyset$ and S itself
- very fast

Remarks

- DPsize/DPsizeLinear does not really test for $p_1 = \epsilon$
- it keeps a list of plans for a given size
- candidates can be found very fast
- ensures polynomial time in some cases (we will look at it again)
- DPsub/DPsubLinear is faster if the problem is not polynomial, though

Memoization

- top-down formulation of dynamic programming
- recursive generation of join trees
- memoize already generated join trees to avoid duplicate work
- easier code
- sometimes more efficient (more knowledge, allows for pruning)
- but usually slower than dynamic programming

Memoization (2)

Memoization(R)
Input: a set of relations $R = \{R_1, \ldots, R_n\}$ to be joined
Output: an optimal bushy join tree
$B =$ an empty DP table $2^R \rightarrow$ join tree
for each $R_i \in R$
  $B[\{R_i\}] = R_i$
MemoizationRec(B, R)
return $B[\{R_1, \ldots, R_n\}]$

- initializes the DP table and triggers the recursive search
- main work done during recursion

Memoization (3)

MemoizationRec(B,S)
Input: a DP table B and a set of relations S to be joined
Output: an optimal bushy join tree for the subproblem
if $B[S] = \epsilon$ {
  for each $S_1 \subset S, S_2 = S \setminus S$
    $p_1 =$MemoizationRec(B, S_1), $p_2 =$MemoizationRec(B, S_2)
    P=CreateJoinTree(p_1, p_2)
    if $B[S] = \epsilon \vee C(B[S]) > C(P)$ $B[S] = P$
  }
}
return $B[S]$

- checks for connectedness omitted