Assertion Level Proof Verification in ΩMEGA

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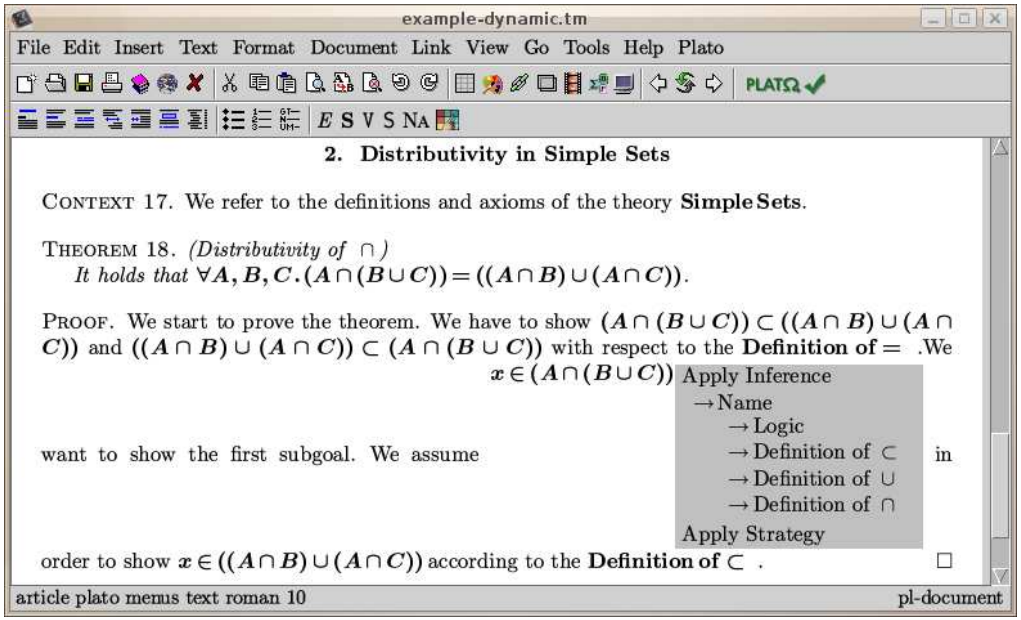
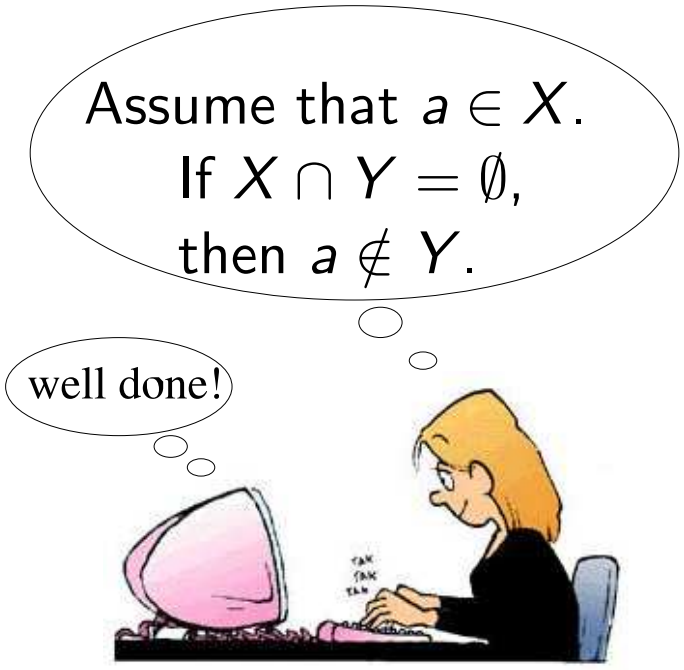
Motivation...

Mathematical Assistance System ΩMEGA is used in different application szenarios:

researcher



tutorial dialog



Task and Problems

Assertion Level Proofs

- Proofs are conducted at or above the **assertion level**
  - Application of lemmata, definitions, theorems, and axioms
- Task: Verify and search such proofs

Problems

- **Gap** between human proof steps and classical calculi
- Some information **underspecified**
  - requires **white box** verification (e.g. underspecified subgoals)
  - leads to **ambiguities**

Idea

- Perform proof search **directly at the assertion level**

Assertion Level Proofs

Example derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\boxed{\forall U, V.\ U \subset V \Leftrightarrow \forall x.x \in U \Rightarrow x \in V}}{\forall V.\ U_1 \subset V_1 \Leftrightarrow \forall x \in U_1 \Rightarrow x \in V_1}}{U_1 \subset V_1 \Leftrightarrow \forall x.x \in U_1 \Rightarrow x \in V_1}}{U_1 \subset V_1 \Rightarrow \forall x.x \in U_1 \Rightarrow x \in V_1} \qquad \boxed{U_1 \subset V_1}}{a_1 \in U_1 \Rightarrow a_1 \in V_1} \qquad \boxed{a_1 \in U_1}}{\boxed{a_1 \in V}}$$

- **Transform** domain knowledge into **proof operators** [AD06]

$$\dfrac{\begin{array}{c}[\,x \in U\,] \\ \vdots \\ x \in V\end{array}}{U \subset V} \qquad \dfrac{x \in U \qquad U \subset V}{x \in V}$$

- Allow for **deep inference**

Verification Algorithm [DB07]

Algorithm

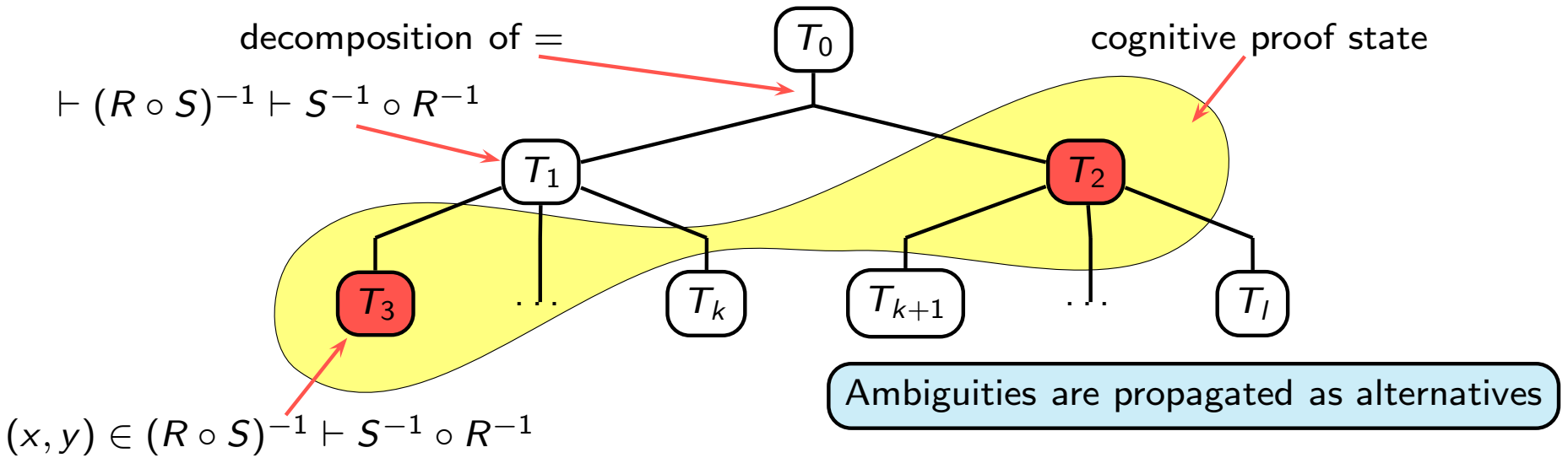
1. Perform a **depth limited BFS**
2. **Filter** consistent successor states
3. **Collect missing subgoals**
4. **Prune** unused nodes

Exercise

$(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

Student Utterance

let $(x, y) \in (R \circ S)^{-1}$



Evaluation [BDSA07]

- 144 **human-level proof steps** by real students
- Proof search **limited to depth 4**

Results

correctly rejected:	28	19 %
correctly accepted:	113	79 %
wrongly accepted:	0	0 %
not verified:	3	2 %

Summary

- Proof step analysis, in particular, correctness analysis, **benefits** from ΩMEGA's **assertion level** proofs
- Simple BFS sufficient to **correctly classify 95.9 %**

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To appear.