



OMEGA
GROUP

$$\begin{aligned} & \sum_{x=0}^{n-1} \sum_{y=0}^{m-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2n}\right) \\ & + \sum_{y=0}^{m-1} s(x, y) \cos\left(\frac{\pi(2y+1)}{2m}\right) \\ & \quad \text{where } s(x, y) = \frac{1}{2} \text{ and } s(0, 0) = 1. \end{aligned}$$

Data Structures in Leo II

Frank Theiß

joint work with C. Benzmüller, L. Paulson, A. Fietzke

March 18, 2008



OMEGA
GROUP

$$I_L = \sum_{m=0}^{L-1} \sum_{n=0}^{L-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2L}\right)$$
$$s(x, y) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{(2x+1)^k}{(2y+1)^k} \cos\left(\frac{\pi(2k+1)}{2L}\right)$$

Leo II

An Automated Theorem Prover

- ▶ resolution based reasoning in HOL
- ▶ compact standalone system implemented in OCaml
- ▶ using efficient Data Structures
- ▶ especially **Term Sharing** and **Term Indexing**



OMEGA
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$$I_L = \int_{-\pi/2}^{\pi/2} \sum_{m=0}^{L-1} \sum_{n=0}^{L-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2L}\right) \sum_{k=0}^{L-1} s(y, k) \cos\left(\frac{\pi(2y+1)}{2L}\right)$$

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OMEGA
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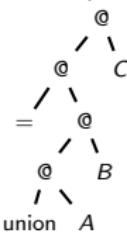
The Problem

Typical data in a resolution based Theorem Prover:

$c_1 \quad l_{1,1} \quad \dots \quad l_{1,m_1}$

\vdots

$c_i \quad l_{i,1} \quad \dots \quad l_{i,j} \dots \quad l_{i,m_i}$



$c_n \quad l_{n,1} \quad \dots \quad l_{i,m_n}$

In a naive implementation:

- ▶ a list of clauses
- ▶ clauses are lists of literals
- ▶ symbols are **string values**
- ▶ terms are **values** of a recursive data structure
 $(appl(t1,t2), abstr(type,t1))$

OMEGA
GROUP

$$\sum_{n=0}^{\infty} s(x, y) \cos\left(\frac{\pi(2x+1)}{2}\right)$$

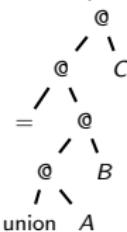
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OMEGA
GROUP

$$\sum_{k=0}^{\infty} \sum_{n=0}^{k-1} \sum_{m=0}^{k-n-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2k}\right)$$
$$= \sum_{y=0}^{\infty} s(x, y) \cos\left(\frac{\pi(2x+1)}{2y}\right)$$

where $x \in \mathbb{R}_{\geq 0}$ and $y \in \mathbb{N}_0$.

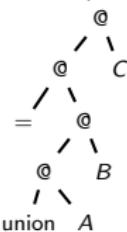
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OMEGA
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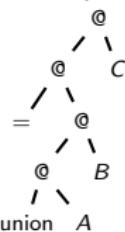
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OMEGA
GROUP

$$\sum_{L=1}^{\infty} \sum_{n=0}^{L-1} \sum_{m=0}^{L-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2L}\right)$$
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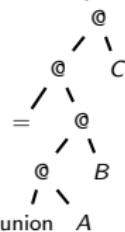
The Problem

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$c_1 \quad l_{1,1} \quad \dots \quad l_{1,m_1}$

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In a naive implementation:

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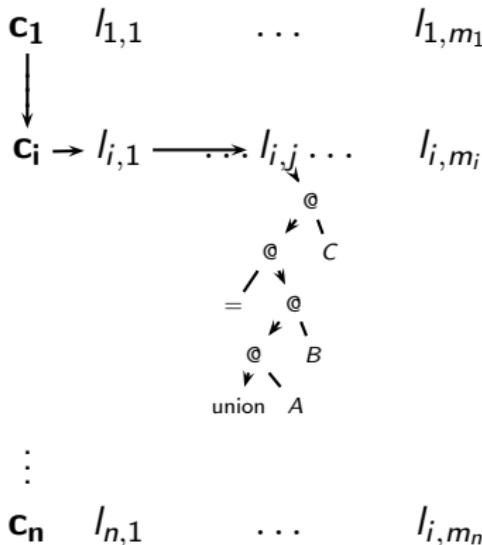
$c_n \quad l_{n,1} \quad \dots \quad l_{n,m_n}$

OMEGA
GROUP

$$\sum_{L=1}^{\infty} \sum_{m=-L}^{L+1} s(x, y) \cos\left(\frac{\pi m x}{2L}\right)$$
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The Problem

Typical data in a resolution based Theorem Prover:



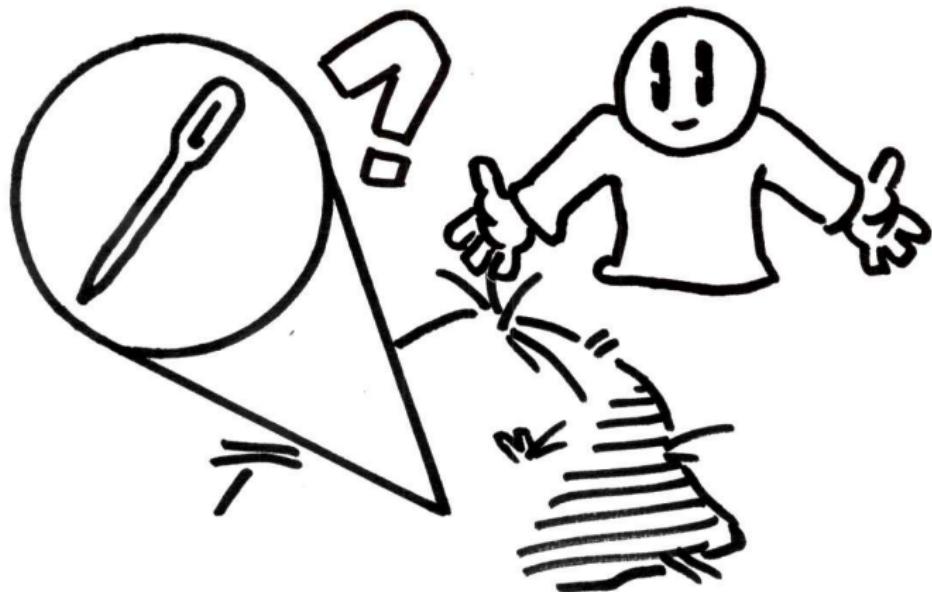
In a naive implementation:

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- ▶ terms as **values** of a recursive data structure (`appl(t1,t2), abstr(type,t1)`)
- ▶ long access paths



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GROUP

A Needle and Haystack Problem...

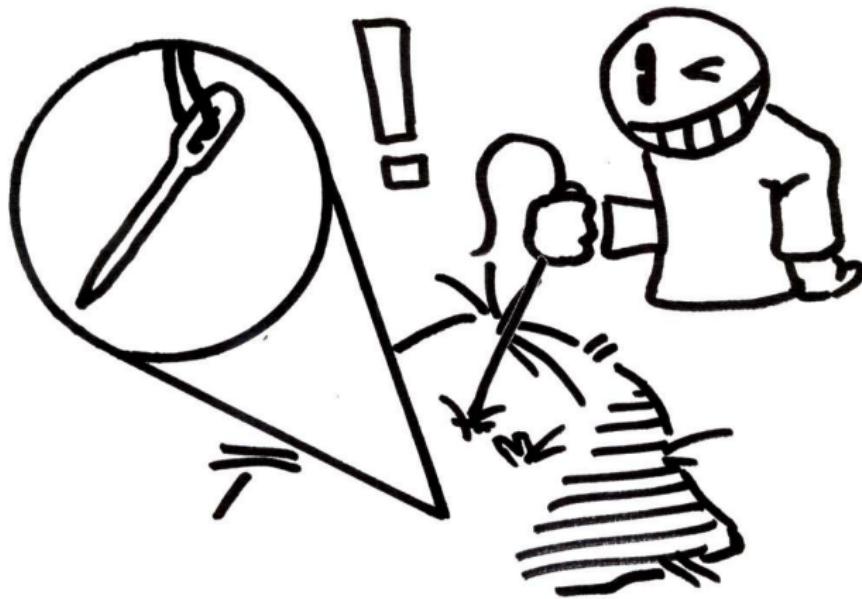




OMEGA
GROUP

$$\begin{aligned} I_L &= \sum_{n=0}^{\infty} \sum_{k=0}^{L-1} S(x, y) \cos\left(\frac{n\pi x}{L}\right) \\ &= \sum_{n=0}^{\infty} S(x, y) \cos\left(\frac{n\pi x}{L}\right) \end{aligned}$$

... with an efficient Solution





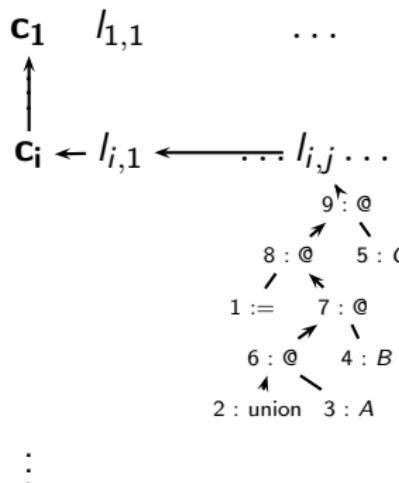
OMEGA
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$$I_L = \sum_{x=0}^{L-1} \sum_{y=0}^{L-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2L}\right)$$

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In Leo II

Terms as graphs:



l_{1,m_1}

l_{i,m_i}

Symbols:

Id	Symbol
1	=
2	union
3	A
4	B
5	C

Applications:

Appl	Func	Arg
6	2	3
7	6	4
8	1	7
9	8	5

Abstractions:

Abstr	Body
:	:
:	:

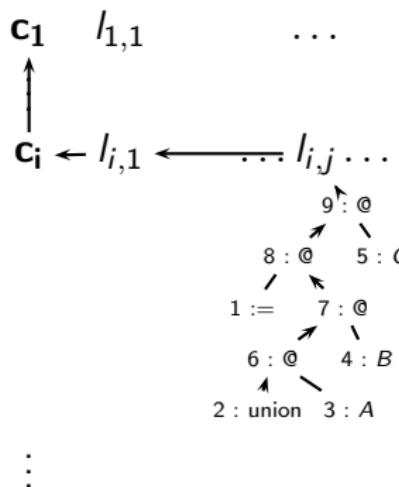


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$$I_L = \int_{-\pi}^{\pi} \sum_{n=0}^{L-1} \sum_{m=0}^{L-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2L}\right) \sum_{y=0}^{L-1} s(x, y) \cos\left(\frac{\pi(2y+1)}{2L}\right)$$

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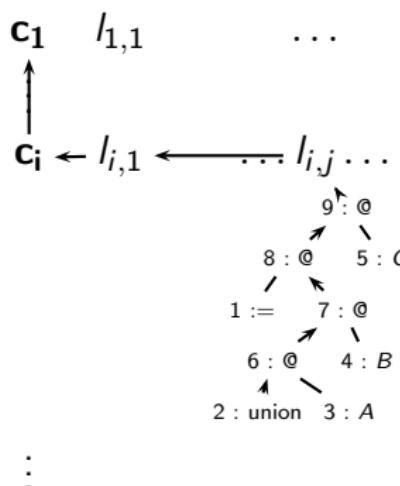


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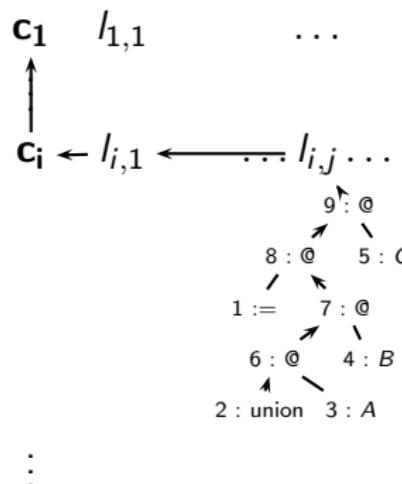
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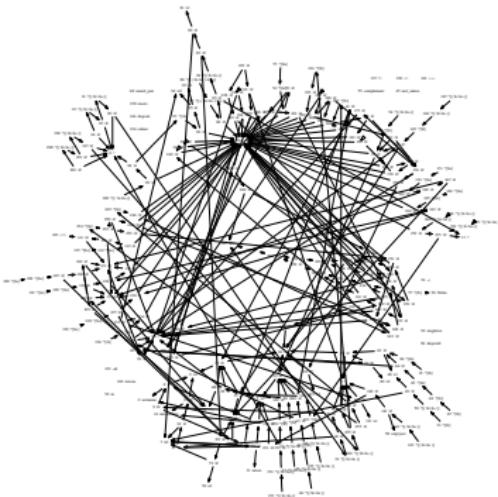


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where $x \in \mathbb{Z}_{\geq 0}$ and $y \in \mathbb{Z}_{\geq 0}$.

Termsharing



In Leo II:

- ▶ Terms as unique instances
- ▶ Perfect Term Sharing
- ▶ Shallow data structures

Adaption to HOL:

- ▶ β - η -normalization
- ▶ DeBruijn indices
- ▶ local contexts for polymorphic type variables

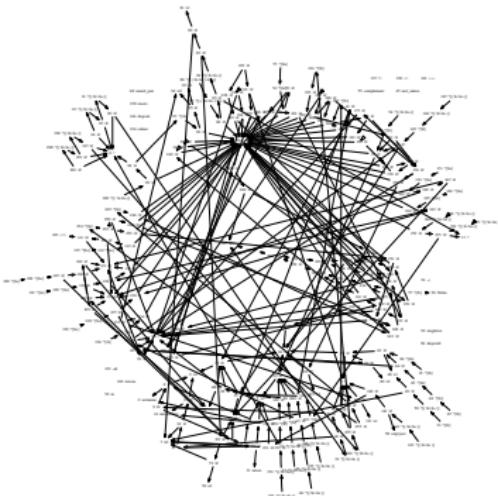


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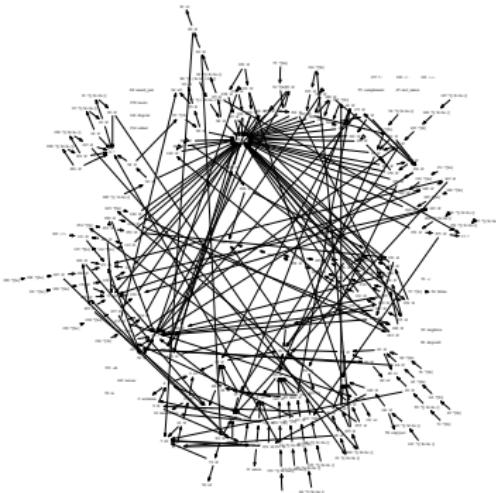


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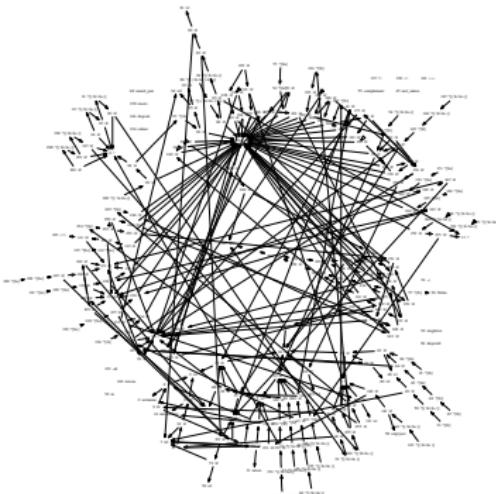


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$$\sum_{L=1}^{L-1} \sum_{x=0}^{2^L-1} \sum_{y=0}^{2^L-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2^L}\right)$$

$$\sum_{y=0}^{2^L-1} s(x, y) \cos\left(\frac{\pi(2x+1)}{2^L}\right)$$

Results

Problem	Vamp. 9.0	LEO+Vamp.	LEO-II+E
014+4	114.5	2.60	0.300
017+1	1.0	5.05	0.059
066+1	—	3.73	0.029
067+1	4.6	0.10	0.040
076+1	51.3	0.97	0.031
086+1	0.1	0.01	0.028
096+1	5.9	7.29	0.033
143+3	0.1	0.31	0.034
171+3	68.6	0.38	0.030
580+3	0.0	0.23	0.078
601+3	1.6	1.18	0.089
606+3	0.1	0.27	0.033
607+3	1.2	0.26	0.036
609+3	145.2	0.49	0.039
611+3	0.3	4.00	0.125
612+3	111.9	0.46	0.030
614+3	3.7	0.41	0.060
615+3	103.9	0.47	0.035
623+3	—	2.27	0.282
624+3	3.8	3.29	0.047
630+3	0.1	0.05	0.025
640+3	1.1	0.01	0.033
646+3	84.4	0.01	0.032
647+3	98.2	0.12	0.037

Problem	Vamp. 9.0	LEO+Vamp.	LEO-II+E
648+3	98.2	0.12	0.037
649+3	117.5	0.25	0.037
651+3	117.5	0.09	0.029
657+3	146.6	0.01	0.028
669+3	83.1	0.20	0.041
670+3	—	0.14	0.067
671+3	214.9	0.47	0.038
672+3	—	0.23	0.034
673+3	217.1	0.47	0.042
680+3	146.3	2.38	0.035
683+3	0.3	0.27	0.053
684+3	—	3.39	0.039
716+4	—	0.40	0.033
724+4	—	1.91	0.032
741+4	—	3.70	0.042
747+4	—	1.18	0.028
752+4	—	516.00	0.086
753+4	—	1.64	0.037
764+4	0.1	0.01	0.032

Vamp. 9.0: 2.80GHz, 1GB memory, 600s time limit

LEO+Vamp.: 2.40GHz, 4GB memory, 120s time limit

LEO-II+E: 1.60GHz, 1GB memory, 60s time limit



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$$= \sum_{y=0}^{\infty} s(x, y) \cos\left(\frac{\pi(2x+y)}{2}\right)$$

Springbreak



2008: Automated Theorem Proving enters the Dancefloor



OMEGA
GROUP

$$\sum_{k=0}^{\infty} \sum_{n=0}^{k-1} \sum_{m=0}^{k-n-1} (-1)^{m+k+n} \frac{(2\pi)^{2k+2}}{2k+2} \frac{(-1)^{k+m+n}}{m+n+1} \frac{1}{(m+1)(m+2)\dots(m+k)} \frac{1}{(n+1)(n+2)\dots(n+k)}$$

Is Automated Reasoning sexy?

