

Sharing is Caring

Combination of Theories

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Deduction at Scale 2011

Outline

- 1 Introduction
 - Combination of Theories
 - Nelson-Oppen
- 2 New Combination Method
 - Arrangements
 - Equality Propagation
 - Care Function
 - Combination Method
- 3 Theory of Uninterpreted Functions
- 4 Experimental Results

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The Theory Combination Problem

Combination of Theories

Given individual decision procedures for (quantifier free) first-order theories T_1 and T_2 how can we combine them in a modular fashion into a decision procedure for a theory $T_1 \oplus T_2$?

- Nelson-Oppen Method (1978)
- Allows one to decide the combination using decision procedures for T_1 and T_2 as black-boxes
- Most SMT Solvers that involve more than one theory use a combination method based on Nelson-Oppen

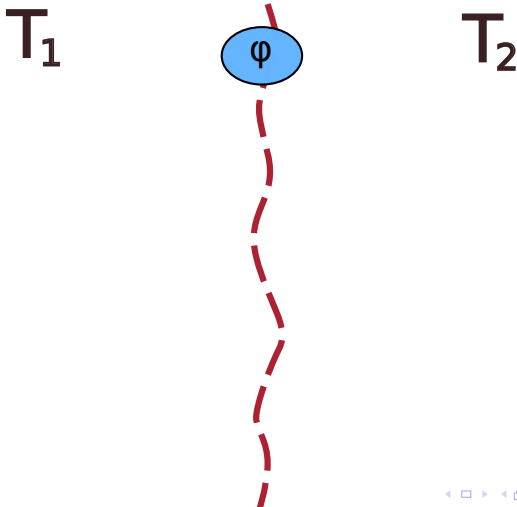
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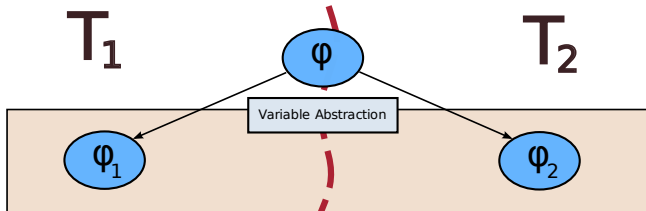
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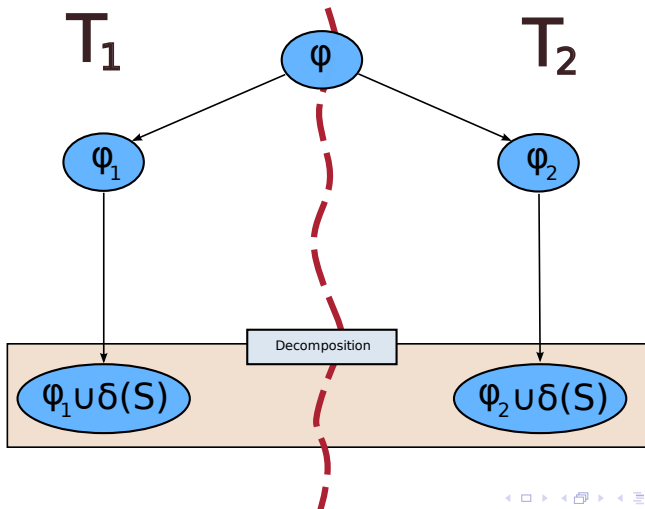
Nelson-Oppen: Idea (1978)



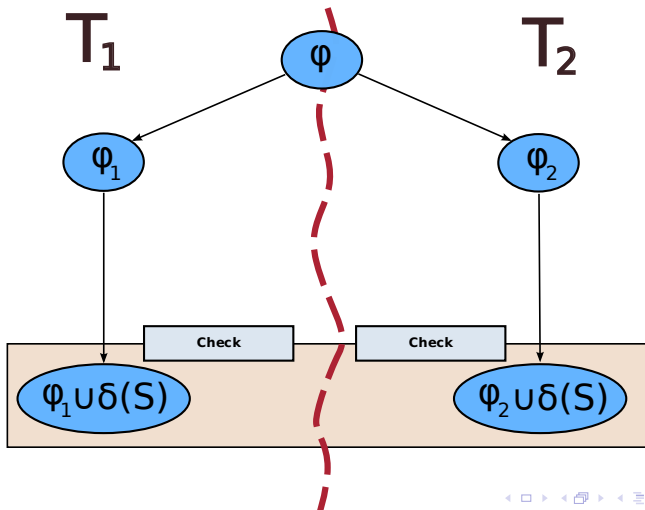
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Nelson-Oppen: Example

$$\bigwedge_{i=1}^n (x_i = f_i(x_{i+1}) \wedge x_i = y_i \times z_i)$$

$$\phi_1 : \bigwedge_{i=1}^n (x_i = f_i(x_{i+1}))$$

$$\phi_2 : \bigwedge_{i=1}^n (x_i = y_i \times z_i)$$

$$S = \{x_1, x_2, \dots, x_n\}$$

$$\phi_1 \wedge \delta(S)$$

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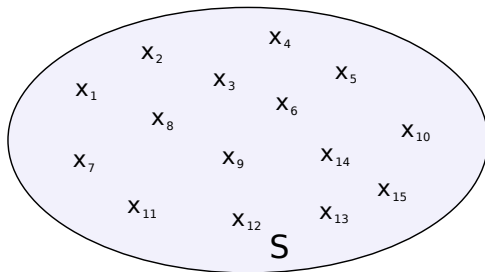
$$\phi_2 \wedge \delta(\mathbf{S})$$

Nelson-Oppen: Complexity

Complexity

Search for a suitable arrangement over the shared variables introduces a heavy layer of complexity:

$$O(\mathcal{T}_1(n)) \oplus O(\mathcal{T}_2(n)) \implies O(2^{n^2} \times (\mathcal{T}_1(n) + \mathcal{T}_2(n))) .$$

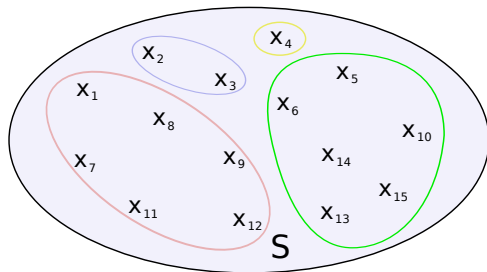


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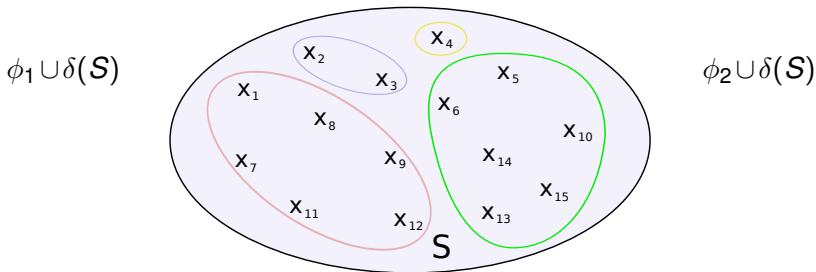


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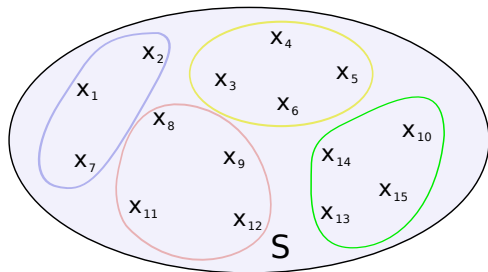


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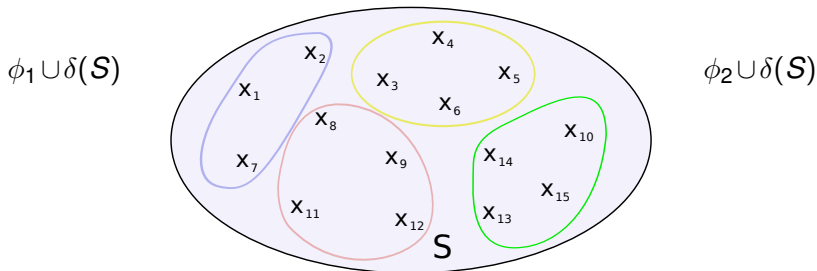


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Nelson-Oppen: Example

But...

We are free to interpret f_i as we wish so there is no need to guess an arrangement.

$$\phi_1 : \bigwedge_{i=1} (x_i = f_i(x_{i+1}))$$

$$\phi_2 : \bigwedge_{i=1} (x_i = y_i \times z_i)$$

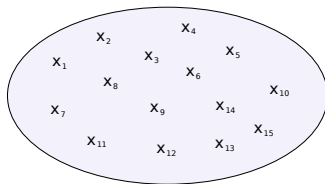
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Arrangements and Care Graphs

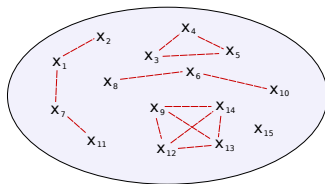
- Instead of guessing an equivalence relation over all the shared variables we will pick the pairs of variables that we (decision procedure) are interested in (care graph).



- We now pick an equivalence relation over the graph G and call the arrangement δ_G .

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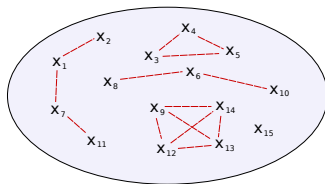
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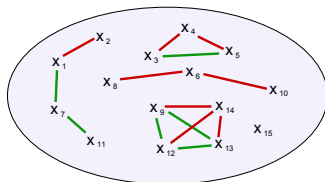
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Equality Propagation

Definition (Equality Propagator $\mathfrak{P}_T^{\equiv}[\cdot]$ for a theory T)

For every set V of variables, maps a set of T -literals ϕ into a set of equalities and dis-equalities between variables in V :

$$\mathcal{E} = \mathfrak{P}_T^{\equiv}[V](\phi) = \{x_1 = y_1, x_2 = y_2, \dots, x_m = y_m\} \cup \{z_1 \neq w_1, z_2 \neq w_2, \dots, z_n \neq w_n\},$$

where $\text{vars}(\mathcal{E}) \subseteq V$ and

- 1 $\phi \implies \mathcal{E}$ is valid in T , and
- 2 $\mathfrak{P}_T^{\equiv}[V]$ is monotone, i.e. $\mathfrak{P}_T^{\equiv}[V](\phi) \subseteq \mathfrak{P}_T^{\equiv}[V](\phi \cup \psi)$.

Equality Propagation

Given two theory propagators:

- 1 $\mathfrak{P}_{T_1}^{\bar{=}}[\cdot]$ for theory T_1
- 2 $\mathfrak{P}_{T_2}^{\bar{=}}[\cdot]$ for theory T_2

we can construct a theory propagator $\mathfrak{P}_T^{\bar{=}}[\cdot]$ for the combined theory $T = T_1 \oplus T_2$

$$\mathfrak{P}_T^{\bar{=}}[V](\phi) = (\mathfrak{P}_{T_1}^{\bar{=}}[V] \oplus \mathfrak{P}_{T_2}^{\bar{=}}[V])(\phi) = \psi_1^* \cup \psi_2^* ,$$

where $\langle \psi_1^*, \psi_2^* \rangle$ is the least fixpoint of the following operator

$$\mathfrak{P}_T^{\bar{=}}[V]\langle \psi_1, \psi_2 \rangle = \langle \mathfrak{P}_{T_1}^{\bar{=}}[V](\phi_1 \cup \psi_2), \mathfrak{P}_{T_2}^{\bar{=}}[V](\phi_2 \cup \psi_1) \rangle .$$

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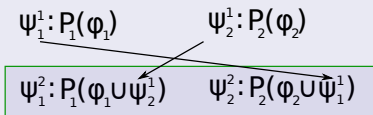
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$$\mathfrak{P}_T^{\equiv}[\mathcal{V}](\phi) = \langle \psi_1^*, \psi_2^* \rangle$$

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$$\mathfrak{P}_T^{\equiv}[\mathcal{V}]\langle \psi_1, \psi_2 \rangle = \langle \psi_1^*, \psi_2^* \rangle$$

Propagation



combined

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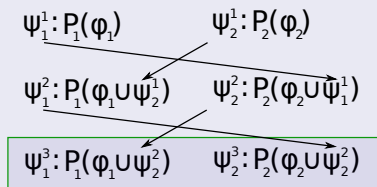
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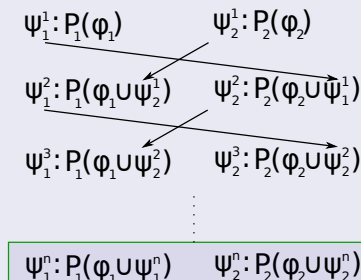
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where $\langle \psi_1^*, \psi_2^* \rangle$ is the least common substitution

$$\mathfrak{P}_T^{\equiv}[\mathcal{V}](\psi_1, \psi_2) = \langle \mathfrak{P}_{T_1}^{\equiv}[\mathcal{V}](\psi_1), \mathfrak{P}_{T_2}^{\equiv}[\mathcal{V}](\psi_2) \rangle$$

Propagation



combined

operator

$\langle \psi_1, \psi_2 \rangle$

Care Function

Definition (Care function $\mathcal{C}[\cdot]$ for T wrt $\mathfrak{P}_T^{\equiv}[\cdot]$)

For a Σ -theory T we call a function $\mathcal{C}[\cdot]$ a *care function* for T with respect to a T -equality propagator $\mathfrak{P}_T^{\equiv}[\cdot]$ when for every set V of variables and every set ϕ of flat Σ -literals

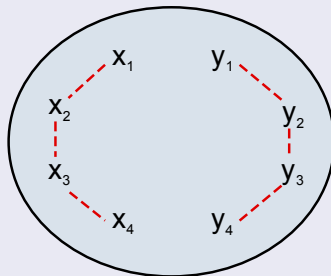
- 1 $\mathcal{C}[V]$ maps ϕ to a care graph $\mathbf{G} = \langle V, E \rangle$;
- 2 if $\mathbf{G} = \langle V, \emptyset \rangle$ and ϕ is T -satisfiable then, for any arrangement δ_V such that $\mathfrak{P}_T^{\equiv}[V](\phi) \subseteq \delta_V$, it holds that $\phi \cup \delta_V$ is also T -satisfiable.

Care Function

Definition (Care function) Care-Function $G = \mathfrak{C}[[V]](\phi)$

For a Σ -theory T we can define the care function with respect to a T -equation set V of variables and an arrangement ϕ :

- 1 $\mathfrak{C}[[V]]$ maps ϕ to a care function G
- 2 if $\mathbf{G} = \langle V, \emptyset \rangle$ and ϕ is an arrangement δ_V such that $\phi \cup \delta_V$ is also T -satisfiable

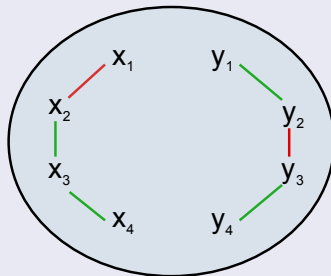


Care Function

Definition (Care function) Decide the equalities in G to get ϕ'

For a Σ -theory T we can decide equalities with respect to a T -equality set V of variables and an arrangement δ_V such that

- 1 $\mathcal{C}[[V]]$ maps ϕ to a care function ϕ'
- 2 if $\mathbf{G} = \langle V, \emptyset \rangle$ and ϕ is an arrangement δ_V such that $\phi \cup \delta_V$ is also T -satisfiable

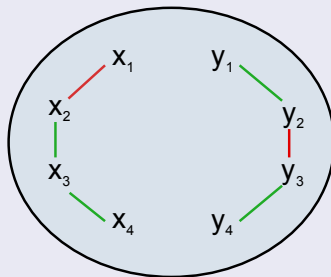


Care Function

Definition (Care function) Care-Function $G' = \mathfrak{C}[[V]](\phi') = \emptyset$

For a Σ -theory T we can work with respect to a T -equation set V of variables and

- 1 $\mathfrak{C}[[V]]$ maps ϕ to a
- 2 if $\mathbf{G} = \langle V, \emptyset \rangle$ and ϕ an arrangement δ_V such that $\phi \cup \delta_V$ is also T -satisfiable

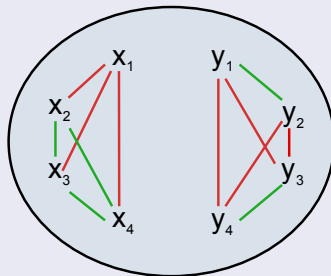


Care Function

Definition (Care function Propagation on ϕ')

For a Σ -theory T we can define care function propagation with respect to a T -equation set V of variables and an arrangement ϕ :

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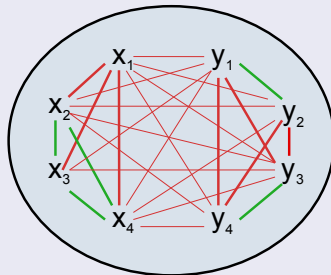


Care Function

Definition (Care function Satisfiable $\phi' \cup \delta_V$)

For a Σ -theory T we can define satisfiability with respect to a T -equation set V of variables and a

- 1 $\mathcal{C}[[V]]$ maps ϕ to a
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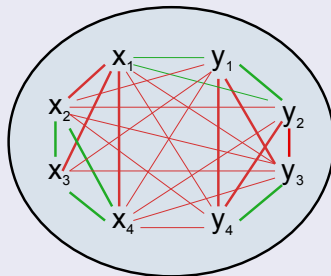


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For a Σ -theory T we call an arrangement ϕ *Care function Satisfiable* with respect to a T -equation set V of variables and δ_V if

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- 2 if $\mathbf{G} = \langle V, \emptyset \rangle$ and ϕ is Care function Satisfiable with respect to V and δ_V then $\phi \cup \delta_V$ is also T -satisfiable

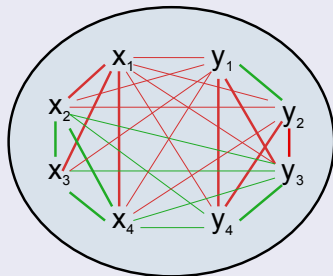


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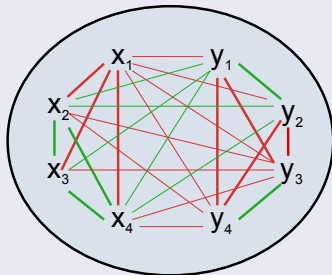


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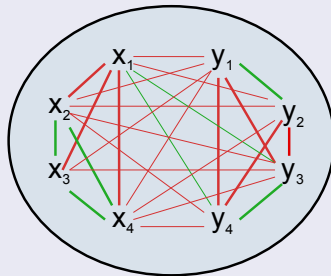


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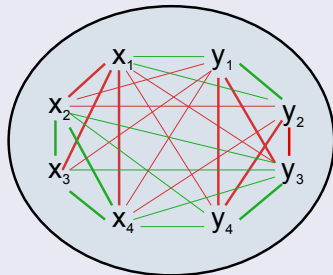


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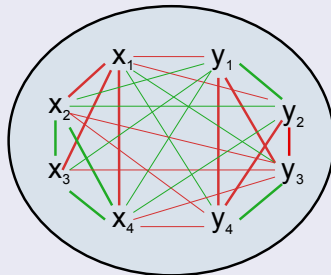


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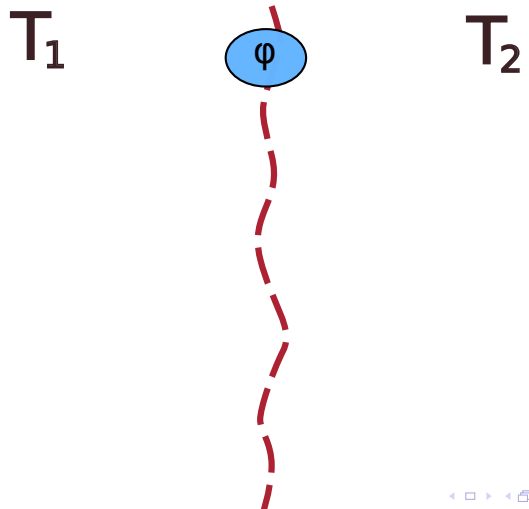
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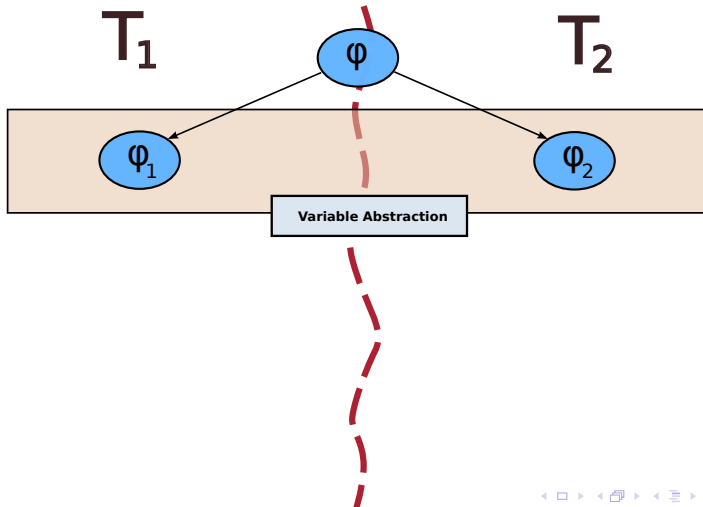
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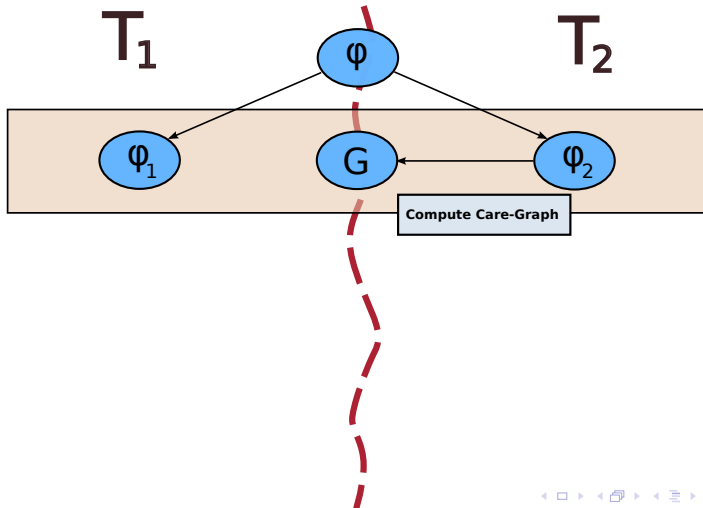
Combination Method



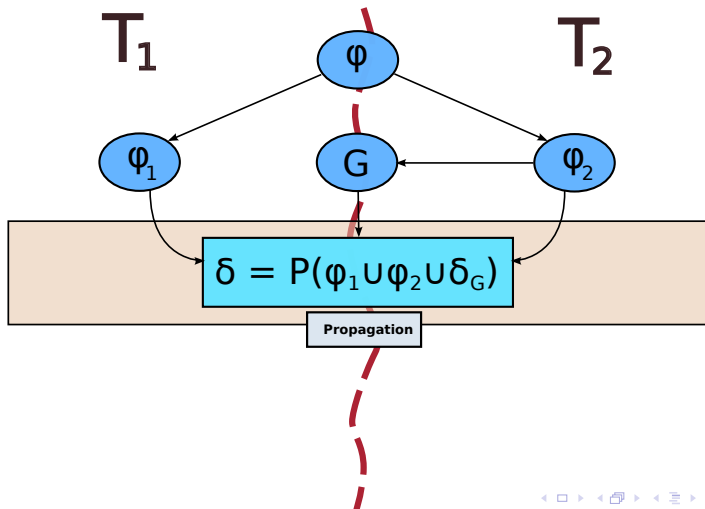
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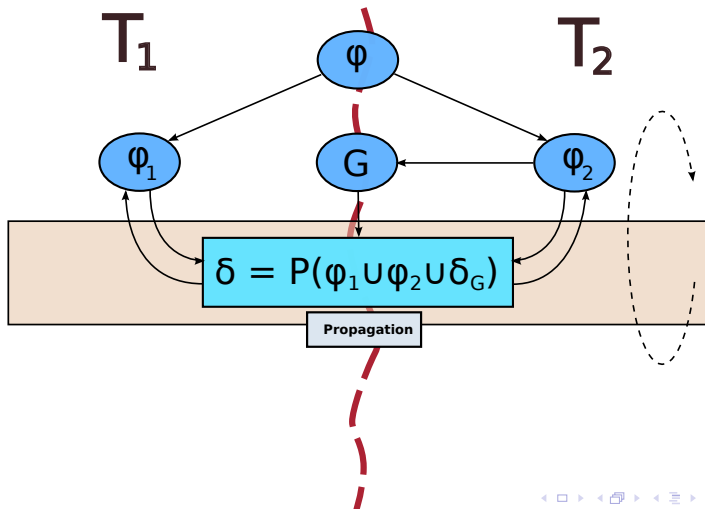
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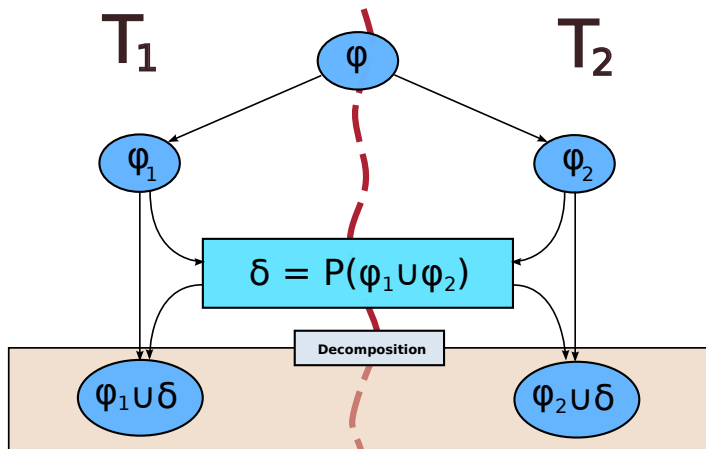
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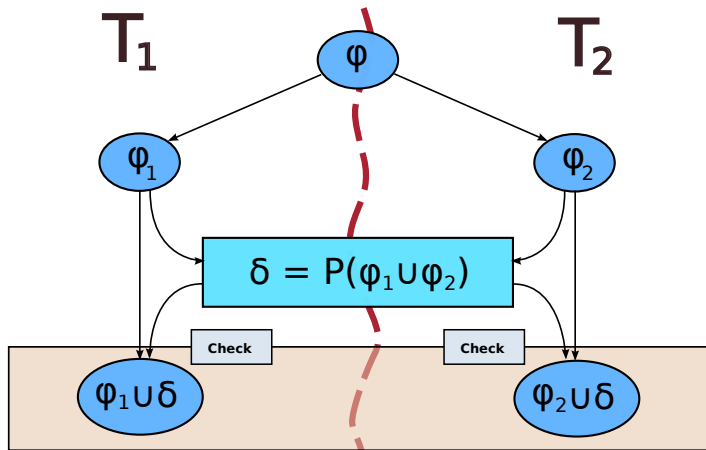
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Combination Method



Combination Method



Theorem

- 1 T_1, T_2 signature-disjoint stably-infinite theory wrt S_1, S_2
- 2 Shared sorts are in $S_1 \cap S_2$
- 3 Care function $\mathcal{C}_{T_2}[\cdot]$ wrt to $\mathfrak{P}_{T_2}^{\equiv}[\cdot]$.

If the QF fragments of T_1 and T_2 are decidable, then the described procedure decides QF fragment of $T_1 \oplus T_2$.

Combination Method

- Equality propagation is explicit in the method
- The new method is asymmetric – only one theory computes the care graph
- Using trivial care functions and propagators the method reduces to standard Nelson-Oppen
- Easily combines with polite theory extension of Nelson-Oppen
- Care functions and propagators need to be devised for the theories

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Theory of Uninterpreted Functions

- Conjunctions of literals the theory can be decided in polynomial time by congruence closure algorithms
- We make use of insights from these algorithms in defining both the equality propagator and the care function
- For simplicity, we assume that the formulas contain no predicate symbols, i.e. literals are

$$x = y, \quad x \neq y, \quad x = f(y_1, \dots, y_n) \ .$$

Equality Propagator

- Let ϕ be a set of literals, V a set of variables, and let \sim_c be the smallest congruence relation over terms in ϕ containing $\{ (x, t) \mid x = t \in \phi \}$.
- We define a dis-equality relation \neq_c as the smallest relation satisfying

$$\begin{aligned}
 x \neq y \in \phi &\implies x \neq_c y , \\
 x \sim_c x' \wedge y \sim_c y' \wedge x' \neq_c y' &\implies x \neq_c y .
 \end{aligned}$$

- We define the equality propagator as

$$\begin{aligned}
 \mathfrak{P}_{\text{euf}}^= \llbracket V \rrbracket (\phi) = & \{ x = y \mid x, y \in V \wedge x \sim_c y \} \cup \\
 & \{ x \neq y \mid x, y \in V \wedge x \neq_c y \} .
 \end{aligned}$$

Equality Propagator

- Let ϕ be a set of literals, V a set of variables, and let \sim_c be the smallest congruence relation over terms in ϕ containing

Example

- Given the set of literals

$$\phi = \{ x = z, y = f(a), z \neq f(a) \} ,$$

the equality propagator $\mathfrak{P}_{\text{euf}}^{\neq}[\cdot]$ would return

- $\mathfrak{P}_{\text{euf}}^{\neq}[[x, y]](\phi) = \{ x = x, y = y, x \neq y, y \neq x \} .$

$$\mathfrak{P}_{\text{euf}}^{\neq}[[V]](\phi) = \{ x = y \mid x, y \in V \wedge x \sim_c y \} \cup \{ x \neq y \mid x, y \in V \wedge x \not\sim_c y \} .$$

Care Function

- Let V be a set of variables and let ϕ be a set of literals
- Assume that ϕ only contains function symbols from

$$F = \{f_1, f_2, \dots, f_n\} .$$

- Using the congruence relation \sim_c from before, for each $f \in F$ let

$$E_f^i = \{ (x_i, y_i) \in V \times V \mid f(\dots, x_i', \dots) \approx_c f(\dots, y_i', \dots) \} ,$$

where $x_i \sim_c x_i'$ and $y_i \sim_c y_i'$, and take the care-graph edges to be $E = \bigcup_{f \in F} \bigcup_{i=1}^{arr(f)} E_f^i$.

- Define the care function as $\mathfrak{C}_{\text{euf}}[\cdot]$

$$\mathfrak{C}_{\text{euf}}[V](\phi) = \mathbf{G} = \langle V, E \rangle .$$

Example

$$V = \{ x_1, x_2, y_2, z_2 \},$$

$$x_1 = x_2 ,$$

$$y_1 = f(x_1) , \quad y_2 = f(x_2) ,$$

$$z_1 = g(x_1) , \quad z_2 = g(x_2) ,$$

$$h(y_1) \neq h(z_1) .$$

- $G_1 = \langle V, (x_1, x_2) \rangle$
- $G_2 = \langle V, (y_2, z_2) \rangle$

Example

$$V = \{ x_1, x_2, y_2, z_2 \},$$

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- $G_1 = \langle V, (x_1, x_2) \rangle$

- $G_2 = \langle V, (y_2, z_2) \rangle$

Example

$$\begin{aligned}V &= \{ x_1, x_2, y_2, z_2 \}, \\x_1 &= x_2, \\y_1 &= f(x_1), \quad y_2 = f(x_2), \\z_1 &= g(x_1), \quad z_2 = g(x_2), \\h(y_1) &\neq h(z_1).\end{aligned}$$

- $G_1 = \langle V, (x_1, x_2) \rangle$
- $G_2 = \langle V, (y_2, z_2) \rangle$

Example

$$\begin{aligned}V &= \{ x_1, x_2, y_2, z_2 \}, \\x_1 &= x_2 , \\y_1 &= f(x_1) , \quad y_2 = f(x_2) , \\z_1 &= g(x_1) , \quad z_2 = g(x_2) , \\h(y_1) &\neq h(z_1) .\end{aligned}$$

- $G_1 = \langle V, (x_1, x_2) \rangle$
- $G_2 = \langle V, (y_2, z_2) \rangle$

- Since the theory of uninterpreted functions is parametrized by the theories that define the function domains (it's using them) it makes sense for it to be in charge of the combination.
- We also have a care-function for the theory of arrays.
- Similar to uninterpreted functions – arrays are can be seen as functions with a twist

Outline

- 1 Introduction
 - Combination of Theories
 - Nelson-Oppen
- 2 New Combination Method
 - Arrangements
 - Equality Propagation
 - Care Function
 - Combination Method
- 3 Theory of Uninterpreted Functions
- 4 Experimental Results

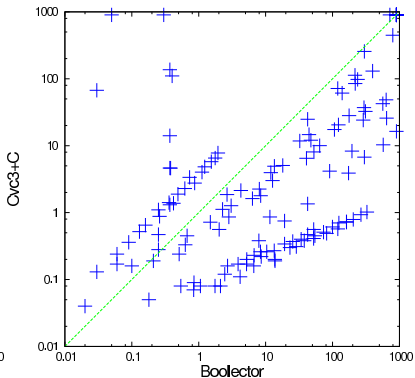
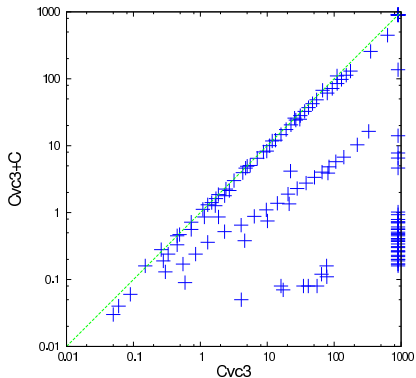
Setup

- We implemented the modified method in Cvc3 and we denote this with Cvc3+C
- The benchmarks are in the QF_AUFBV (arrays+bit-vectors)
- Benchmarks are SMT-LIB problems with more than one array variable
- Compared to Boolector, Yices, MathSAT, Z3, and Cvc3 (includes all solvers from 2009 SMT-COMP competition).

Experimental Results

	Boolector	Yices	MathSAT	Z3	Cvc3	Cvc3+C
crafted (40)	2100 (40)	6253 (34)	468 (30)	112 (40)	388 (9)	14 (40)
matrix (11)	1208 (10)	683 (6)	474 (4)	927 (11)	831 (11)	45 (11)
unconstr (10)	3 (10)	(0)	706 (3)	54 (2)	185 (5)	340 (8)
copy (19)	11 (19)	1 (19)	1103 (19)	4 (19)	432 (17)	44 (19)
sort (6)	691 (6)	557 (4)	82 (4)	248 (3)	44 (6)	44 (6)
delete (29)	3407 (18)	1170 (10)	2626 (14)	1504 (10)	1766 (17)	1302 (17)
member (24)	2807 (24)	185 (24)	217 (24)	112 (24)	355 (24)	320 (24)
	10229 (127)	8852 (97)	5678 (98)	2965 (109)	4004 (89)	2112 (125)

Experimental Results



Conclusion

- Reformulation of the classic Nelson-Oppen method for combining theories.
- Reduces the complexity of finding a common arrangement over the interface variables.
- We devised care functions for the theories of uninterpreted functions and arrays.
- The new method is asymmetric.
- Orthogonal to previous research on combinations of theories.
- Robust performance increase over the standard Nelson-Oppen implementation.

Thank you!

