# Model Checking of Action-Based Concurrent Systems

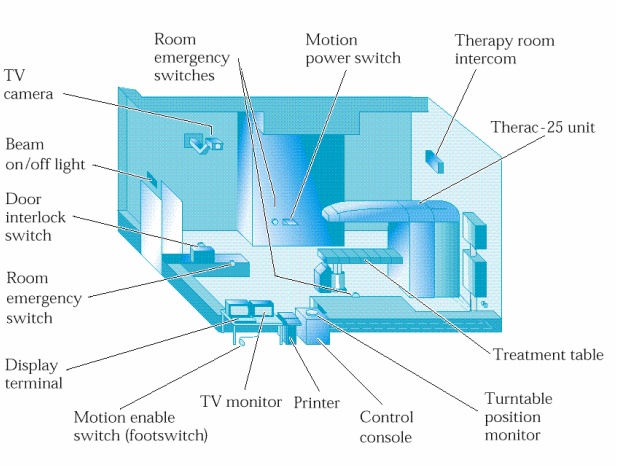
**Radu Mateescu**

*INRIA Rhône-Alpes / VASY*

http://www.inrialpes.fr/vasy



# Why formal verification?





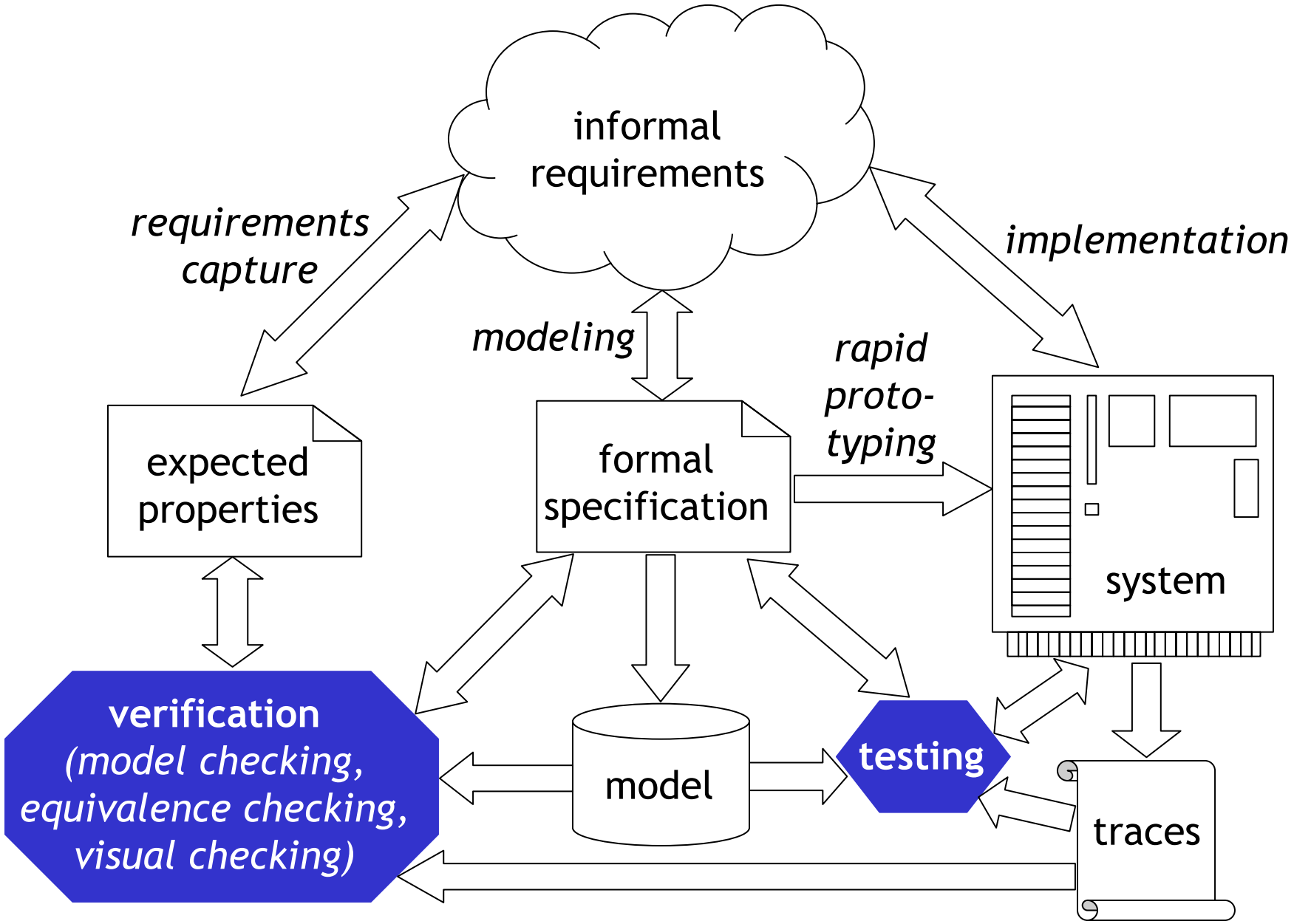
Therac-25 radiotherapy accidents (1985-1987)

Ariane-5 launch failure (1996)

Mars climate orbiter failure (1999)

- Characteristics of these systems
  - Errors due to software
  - Complex, often involving parallelism
  - Safety-critical

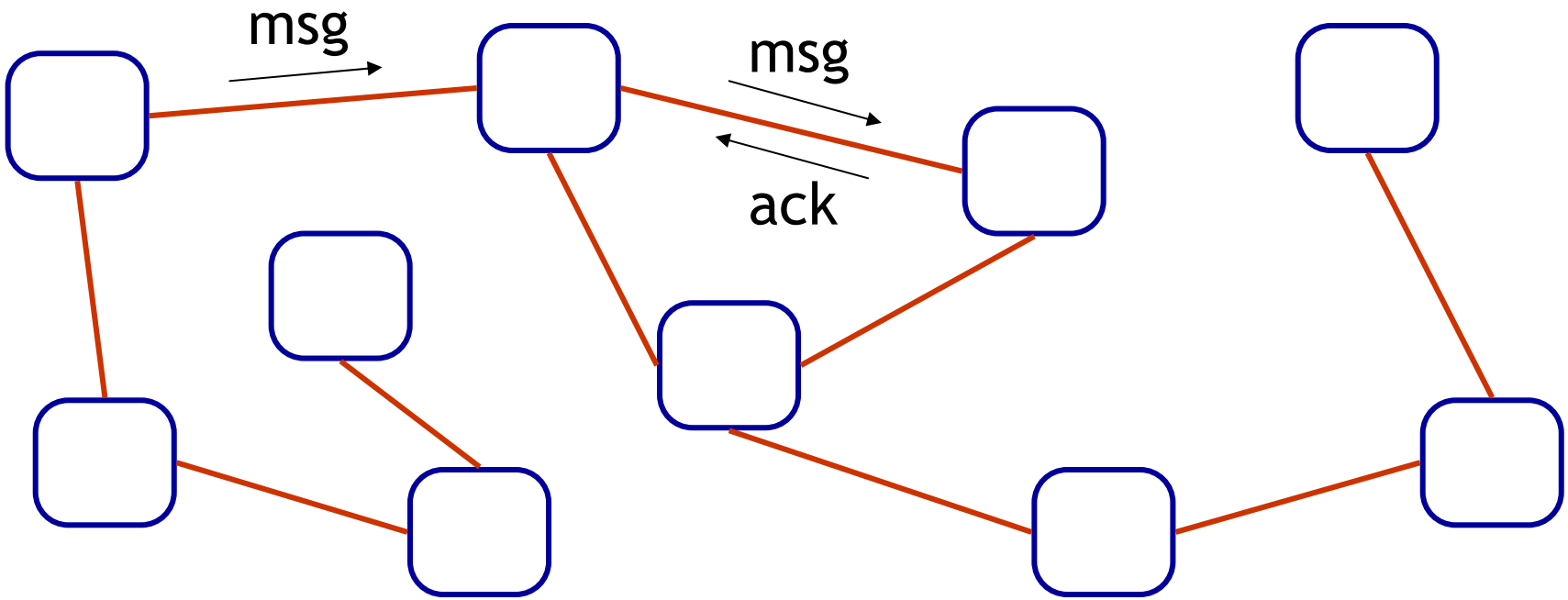
➔ *formal verification is useful for early error detection*


informal
requirements
requirements
capture
modeling
implementation
rapid
proto-
typing
expected
properties
formal
specification
system
verification
(model checking,
equivalence checking,
visual checking)
model
testing
traces

# Outline

- Communicating automata
- Process algebraic languages
- Action-based temporal logics
- On-the-fly verification
- Case study
- Discussion and perspectives

# Asynchronous concurrent systems




## Characteristics:

- Set of distributed processes
- Message-passing communication
- Nondeterminism

## Applications:

- Hardware
- Software
- Telecommunications

# CADP toolbox: Construction and Analysis of Distributed Processes

(http://www.inrialpes.fr/vasy/cadp)

- Description languages:
  - ISO standards (LOTOS, E-LOTOS)
  - Networks of communicating automata
- Functionalities:
  - Compilation and rapid prototyping
  - Interactive and guided simulation
  - Equivalence checking and model checking
  - Test generation
- Case-studies and applications:
  - >100 industrial case-studies
  - >30 derived tools
- Distribution: over 400 sites (2008)

# Communicating automata

- Basic notions
- Implicit and explicit representations
- Parallel composition and synchronization
- Hiding and renaming
- Behavioural equivalences

# Transformational systems

- Work by computing a result in function of the entries
- Absence of termination undesirable
- Upon termination, the result is unique
- Sequential programming (sorting algorithms, graph traversals, syntax analysis, ...)

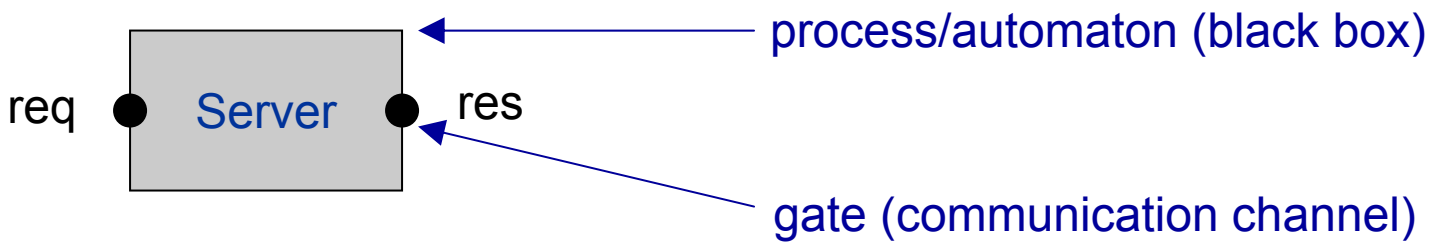
# Reactive systems

- Work by reacting to the stimuli of the environment
- Absence of termination desirable
- Different occurrences of the same request may produce different results
- Parallel programming (operating systems, communication protocols, Web services, ...)

- Concurrent execution
- Communication + synchronization

# Communicating automata

- Simple formalism describing the behaviour of concurrent systems
- *Black-box* approach:
  - One cannot inspect directly the state of the system
  - The behaviour of the system can be known only through its interactions with the environment




- Synchronization on a gate requires the participation of the process and of its environment (*rendezvous*)

# Automaton (LTS)

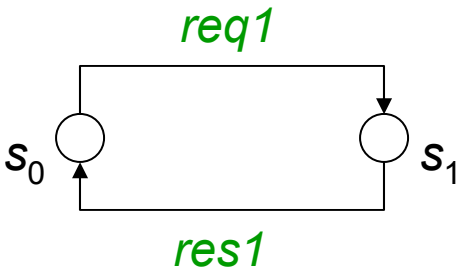
- Labeled Transition System $M = \langle S, A, T, s_0 \rangle$
  - $S$: set of *states* ($s_1$, $s_2$, ...)
  - $A$: set of visible *actions* ($a_1$, $a_2$, ...)
  - $T$: *transition* relation ($s_1 - a \rightarrow s_2 \in T$)
  - $s_0 \in S$: *initial state*

*internal action (noted i or $\tau$ )*

*every state is reachable from the initial state*

*deadlock (sink) state: no outgoing transitions*

- Example: process $client_1$




*sequential model of a reactive system behaviour*

- Other kinds of automata:
  - Kripke strictures (information associated to states)
  - Input/output automata [Lynch-Tuttle]

# LTS representations in CADP

(http://www.inrialpes.fr/vasy/cadp)

## Explicit

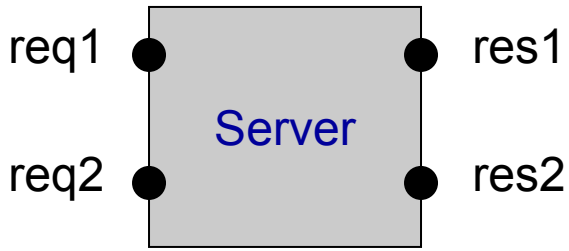
- List of transitions
- Allows forward and backward exploration
- Suitable for global verification
- BCG (Binary Coded Graphs) environment
  - API in C for reading/writing
  - Tools and libraries for explicit graph manipulation (**bcg_**io, **bcg_**draw, **bcg_**info, **bcg_**edit, **bcg_**labels, ...)
  - Global verification tools (XTL)

## Implicit

- "Successor" function
- Allows forward exploration only
- Suitable for local (or on-the-fly) verification
- Open/Caesar environment [Garavel-98]
  - API in C for LTS exploration
  - Libraries with data structures for implicit graph manipulation (stacks, tables, edge lists, hash functions, ...)
  - On-the-fly verification tools (Bisimula**tor**, Evalua**tor**, ...)

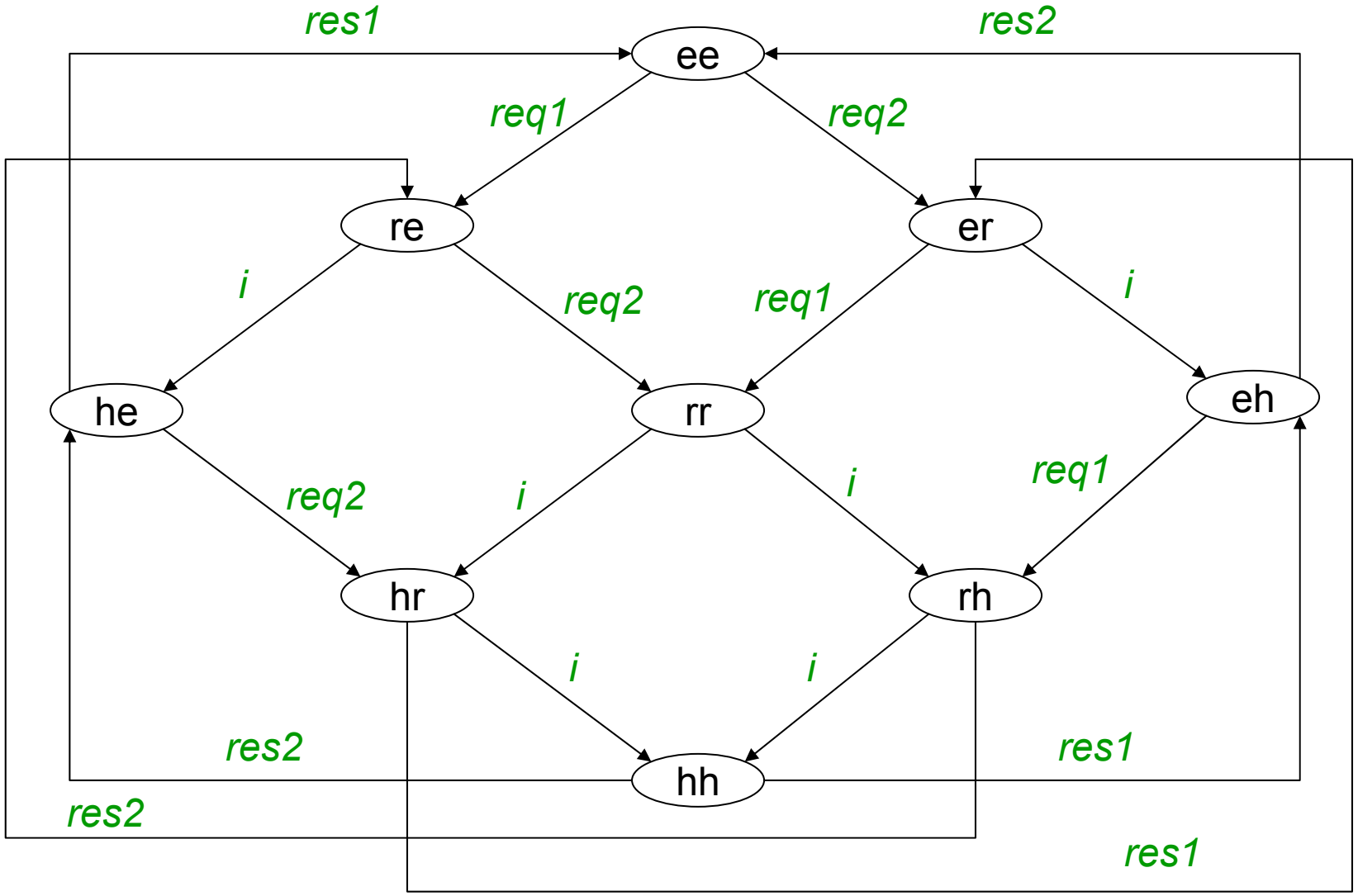
# Server example
## (modeled using a single automaton)

- Server able to process two requests concurrently
- State variables $u_1$, $u_2$ storing the request status:
  - Empty (e)
  - Received (r)
  - Handled (h)
- A state: couple <$u_1$, $u_2$>
- Initial state: <e, e> (ee for short)
- Gates (actions):
  - req1, req2: receive a request
  - res1, res2: send a response
  - i: internal action



# LTS of the server

(9 states, 16 transitions)



# Remarks

- All the theoretical states are reachable:

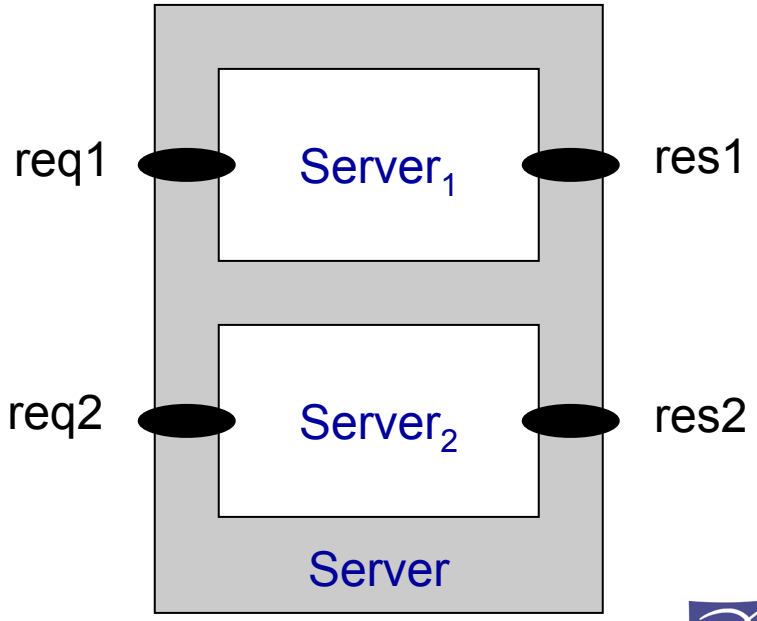
  $| u_1 | * | u_2 | = 3 * 3 = 9$

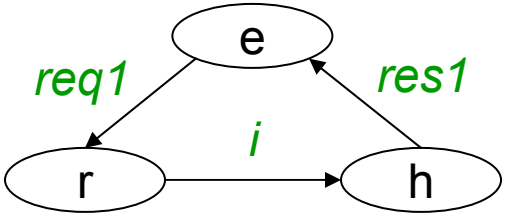
  (no synchronization between request processings)
- There is no sink state (the system is *deadlock-free*)
- From every state, it is possible to reach the initial state again (the server can be re-initialized)
- Shortcomings of modeling with a single automaton:
  - One must predict all the possible request arrival orders
  - For more complex systems, the LTS size grows rapidly

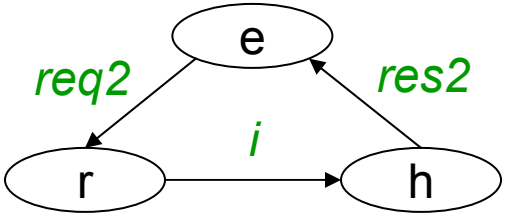
➔ *need of higher-level modeling features*

# Server example
## (modeled using two concurrent automata)

- Decomposition of the system in two subsystems
  - Every type of request is handled by a subsystem
  - In the server example, subsystems are independent
- Simpler description w.r.t. single automaton:
  3 + 3 = 6 states



# Decomposition in concurrent subsystems

## Required at physical level

- Modeling of distributed activities
- Multiprocessor/multitasking execution platform

## Chosen at logical level

- Simplified design of the system
- Well-structured programs

- Communication and synchronization between subsystems may introduce behavioural errors (e.g., *deadlocks*)
- In practice, even simple parallel programs may reveal difficult to analyze

➔ *need of computer-assisted verification*

# Parallel composition ("product") of automata

- Goals:
  - Define internal composition laws

    $$\otimes : \text{LTS} \times \ldots \times \text{LTS} \rightarrow \text{LTS}$$

    expressing the parallel composition of 2 (or more) LTSs
  - Allow synchronizations on one or several actions (gates)
  - Allow hierarchical decomposition of a system
- Consequences:
  - A product of automata can always be translated into a single (sequential) automaton
  - The logical parallelism can be implemented sequentially (e.g., time-sharing OS)

# Binary parallel composition
(syntax)

- EXP language [Lang-05]
  - Description of communicating automata
  - Extensive set of operators
    - Parallel compositions (binary, general, ...)
    - Synchronization vectors
    - Hiding / renaming, cutting, priority, ...
  - Exp.Open compiler ➔ implicit LTS representation
- Binary parallel composition:

"lts1.bcg" |[G1, ..., Gn]| "lts2.bcg" — with synchronization on G1, ..., Gn

"lts1.bcg" ||| "lts2.bcg" — without synchronization (interleaving)

# Binary parallel composition
## (semantics)

Let $M_1 = \langle S_1, A_1, T_1, s_{01} \rangle$, $M_2 = \langle S_2, A_2, T_2, s_{02} \rangle$ and $L \subseteq A_1 \cap A_2$ a set of visible actions to be synchronized.

$M_1 \; |[ \; L \; ]| \; M_2 = \langle S, A, T, s_0 \rangle$

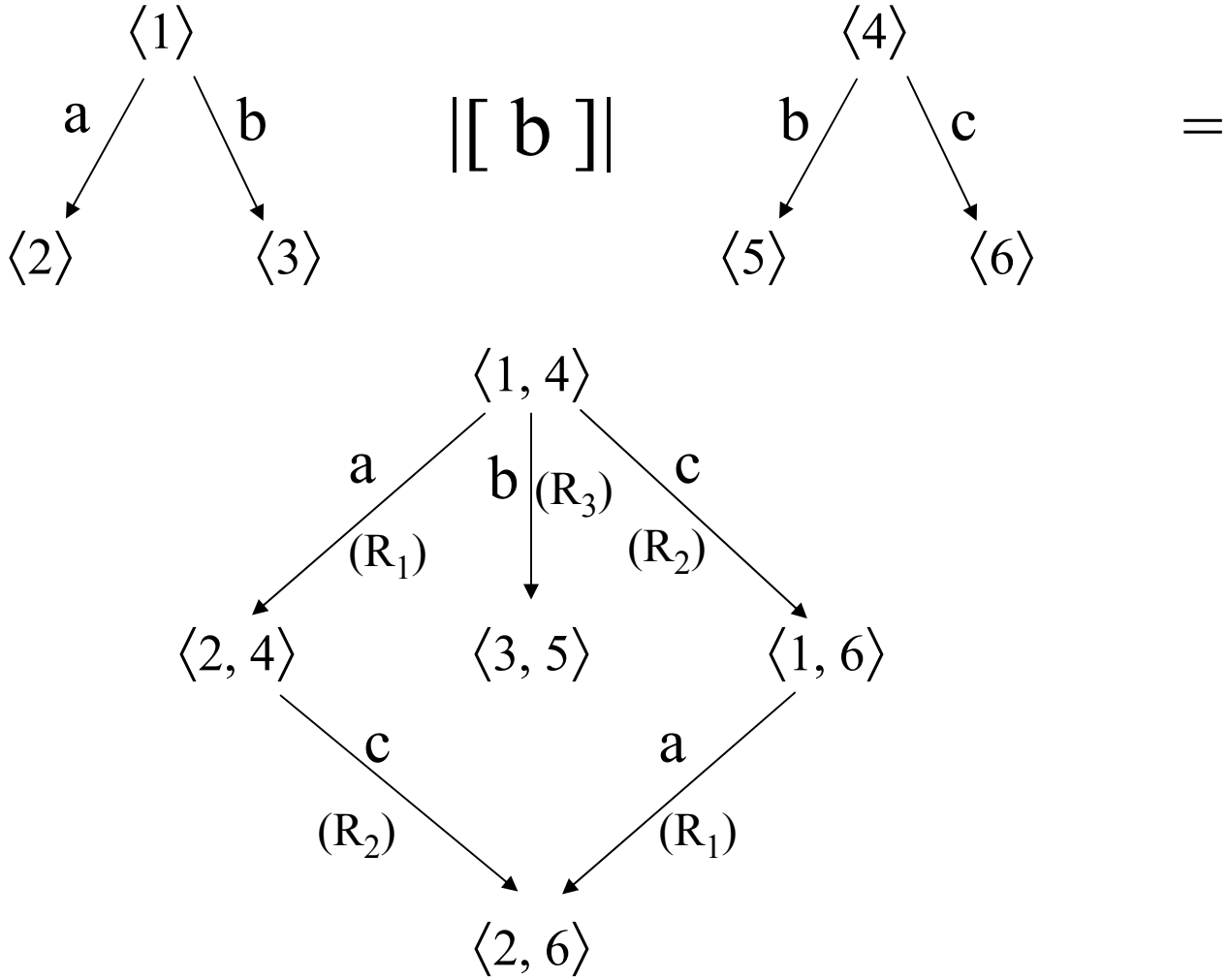
- $S = S_1 \times S_2$
- $A = A_1 \cup A_2$
- $s_0 = \langle s_{01}, s_{02} \rangle$
- $T \subseteq S \times A \times S$ defined by $R_1$-$R_3$

$$(R_1) \quad \frac{s_1 \xrightarrow{a} s'_1 \;\wedge\; a \notin L}{\langle s_1, s_2 \rangle \xrightarrow{a} \langle s'_1, s_2 \rangle}$$

$$(R_2) \quad \frac{s_2 \xrightarrow{a} s'_2 \;\wedge\; a \notin L}{\langle s_1, s_2 \rangle \xrightarrow{a} \langle s_1, s'_2 \rangle}$$

$$(R_3) \quad \frac{s_1 \xrightarrow{a} s'_1 \wedge s_2 \xrightarrow{a} s'_2 \;\wedge\; a \in L}{\langle s_1, s_2 \rangle \xrightarrow{a} \langle s'_1, s'_2 \rangle}$$

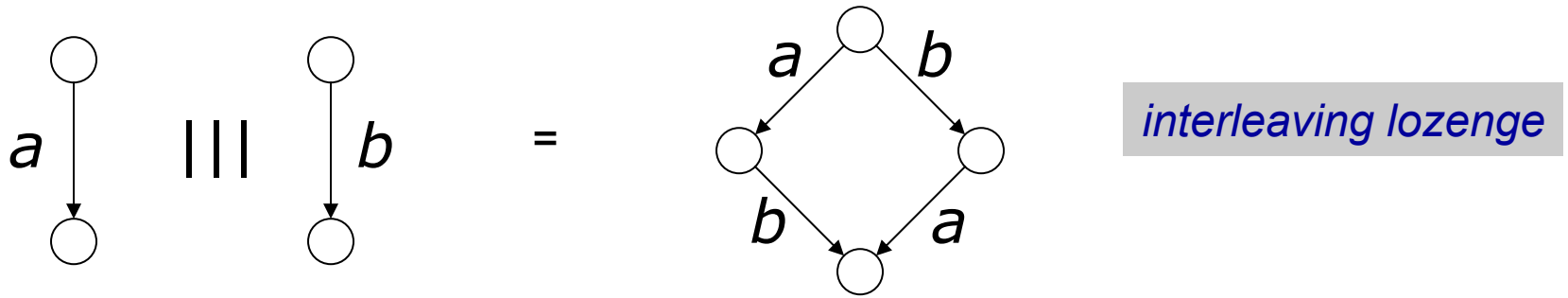
# Example



# Interleaving semantics

- Hypothesis:
  - Every action is atomic
  - One can observe at most one action at a time

➔ *suitable paradigm for distributed systems*

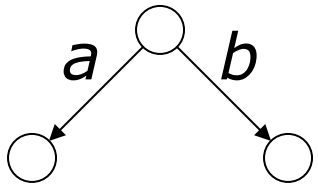



*interleaving lozenge*

- Parallelism can be expressed in terms of *choice* and *sequence* (*expansion theorem* [Milner-89])

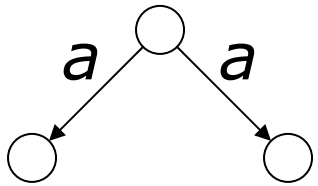
# Internal and external choice

- *External* choice (the environment decides which branch of the choice will be executed)




*the environment can force the execution of a and b by synchronizing on that action*

- *Internal* choice (the system decides)




*the environment may synchronize on a, but this will not remove the nondeterminism*

# Example of modeling with communicating automata

- Mutual exclusion problem:

  *Given two parallel processes $P_0$ and $P_1$ competing for a shared resource, guarantee that at most one process accesses the resource at a given time.*

- Several solutions were proposed *at software level*:
  - In centralized setting (Peterson, Dekker, Knuth, ...)
  - In distributed setting (Lamport, ...)

➔ M. Raynal. *Algorithmique du parallélisme: le problème de l'exclusion mutuelle.* Dunod Informatique, 1984.

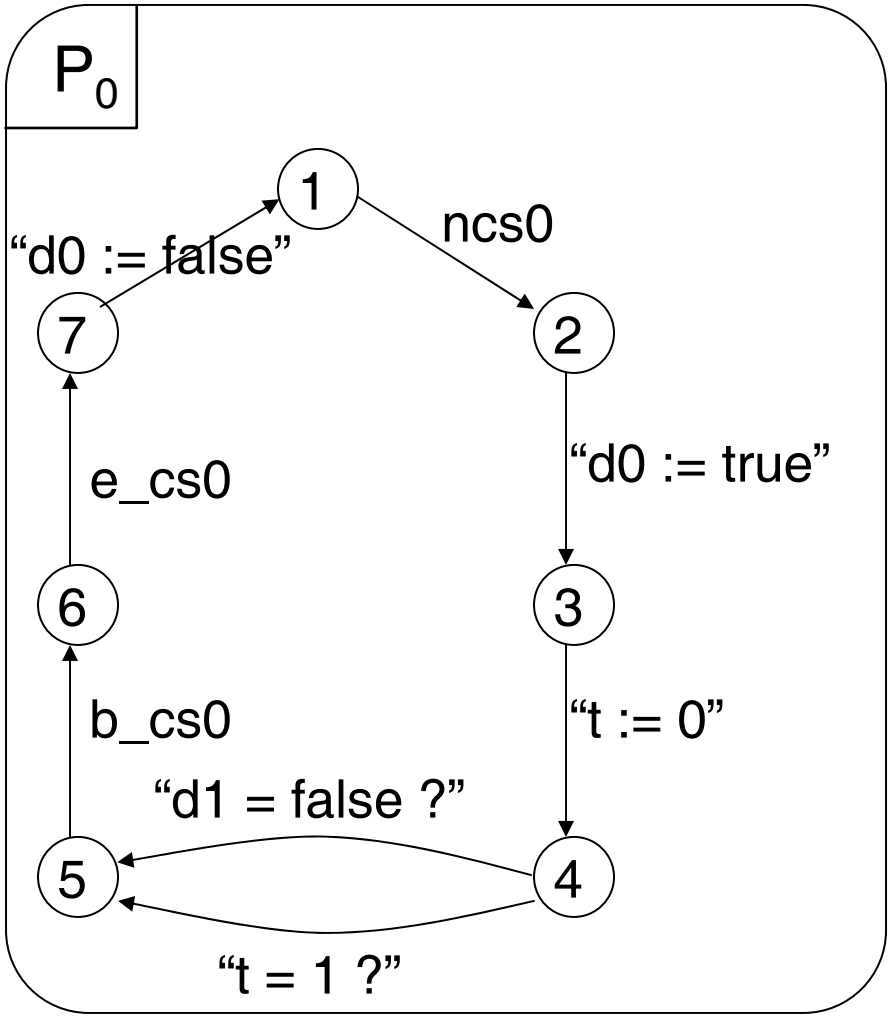
# Peterson's algorithm [1968]

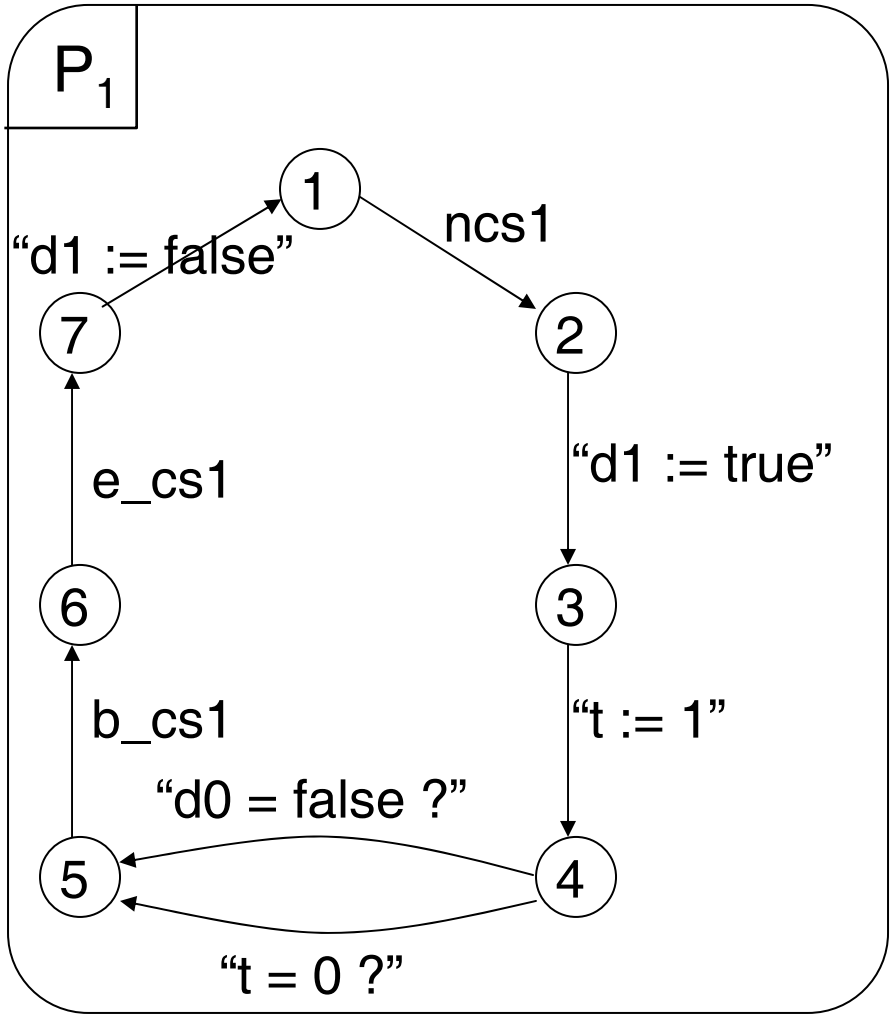
**var** d0 : bool := false { read by P1, written by P0 }
**var** d1 : bool := false { read by P0, written by P1 }
**var** t ∈ {0, 1} := 0 { read/written by P0 and P1 }

**loop forever** { P0 }
1 : { ncs0 }
2 : d0 := true
3 : t := 0
4 : **wait** (d1 = false **or** t = 1)
5 : { b_cs0 }
6 : { e_cs0 }
7 : d0 := false
**endloop**

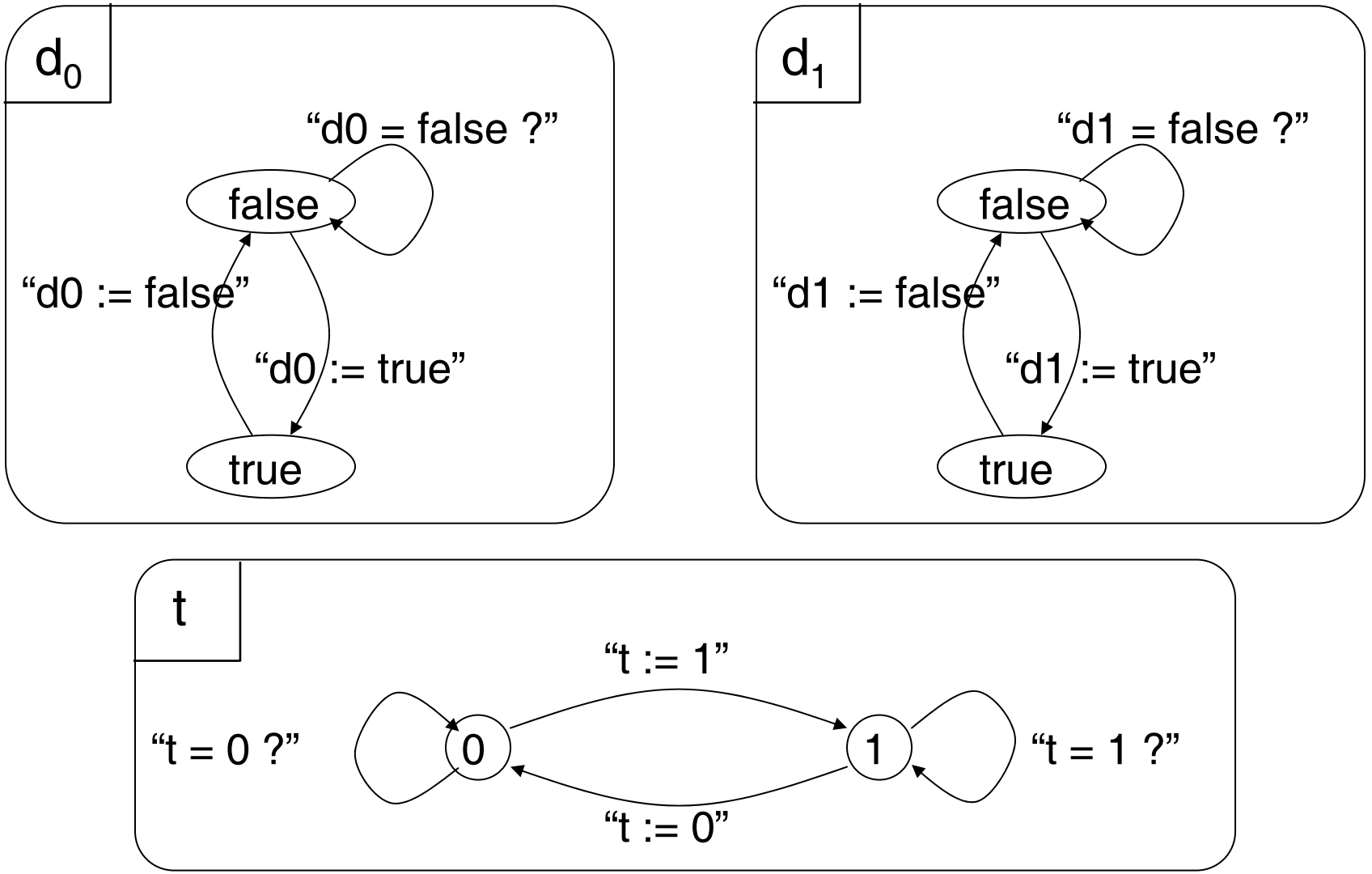
**loop forever** { P1 }
1 : { ncs1 }
2 : d1 := true
3 : t := 1
4 : **wait** (d0 = false **or** t = 0)
5 : { b_cs1 }
6 : { e_cs1 }
7 : d1 := false
**endloop**

# Automata of $P_0$ and $P_1$

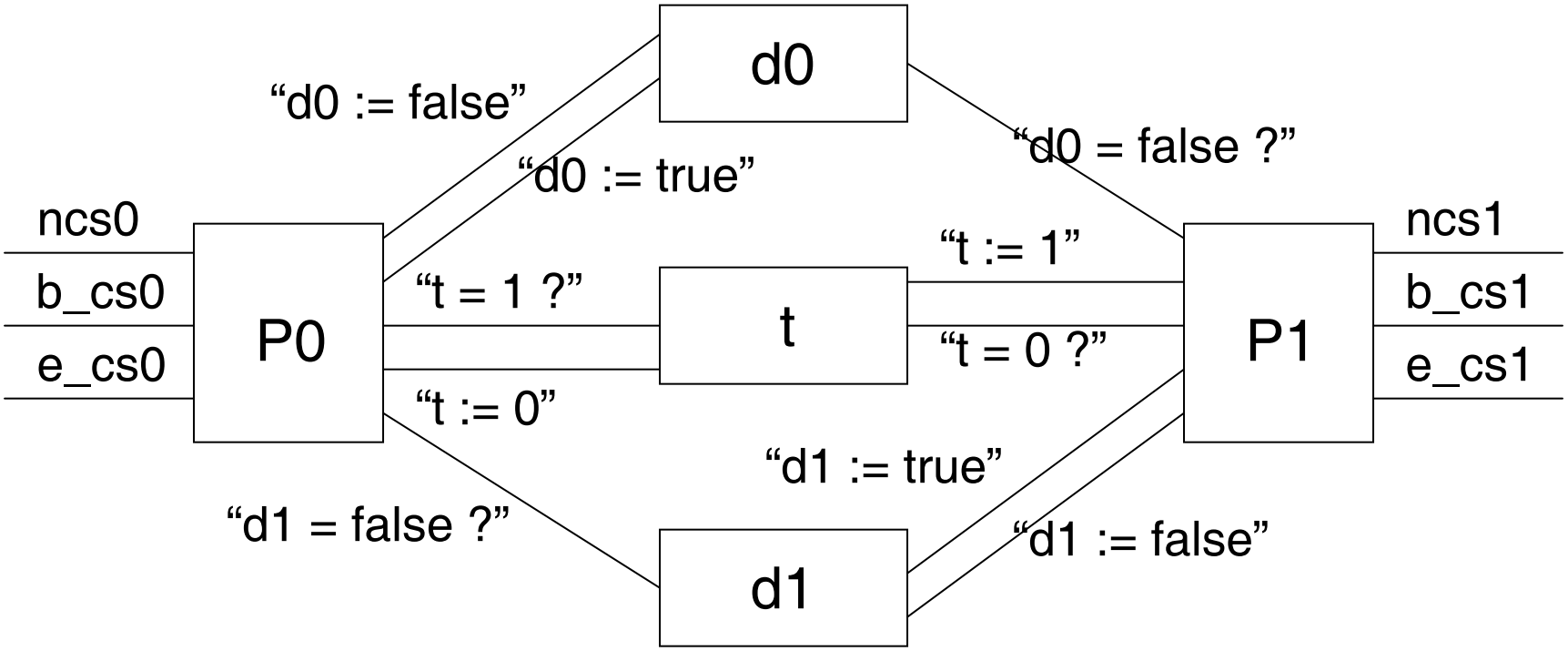


# Automata of $d_0$, $d_1$, and t



# Architecture of the system
(graphical)




- Synchronized actions: «d0:=false», «d0:=true», ...
- Non synchronized actions: ncs0, b_cs0, e_cs0, ...

# Architecture of the system
## (textual)

- Using binary parallel composition:

  (P0 ||| P1)
  |[ “d0:=false”, “d0:=true”, ... ]|
  (d0 ||| d1 ||| t)

- Using general parallel composition:

  **par**
  “d0:=false”, “d0:=true”, ... → P0
  || “d1:=false”, “d1:=true”, ... → P1
  || “d0:=false”, “d0:=true”, “d0=false?” → d0
  || “d1:=false”, “d1:=true”, “d1=false?” → d1
  || “t:=0”, “t:=1”, “t=0?”, “t=1?” → t
  **end par**

# Construction of the LTS
## (“product automaton”)

- *Explicit-state* method:
  - LTS construction by exploring forward the transition relation, starting at the initial state
  - Transitions are generated by using the $R_1$, $R_2$, $R_3$ rules
  - Detect already visited states in order to avoid cycling
- Several possible exploration strategies:
  - Breadth-first, depth-first
  - Guided by a criterion / property, ...
- Several types of algorithms:
  - Sequential, parallel, distributed, ...

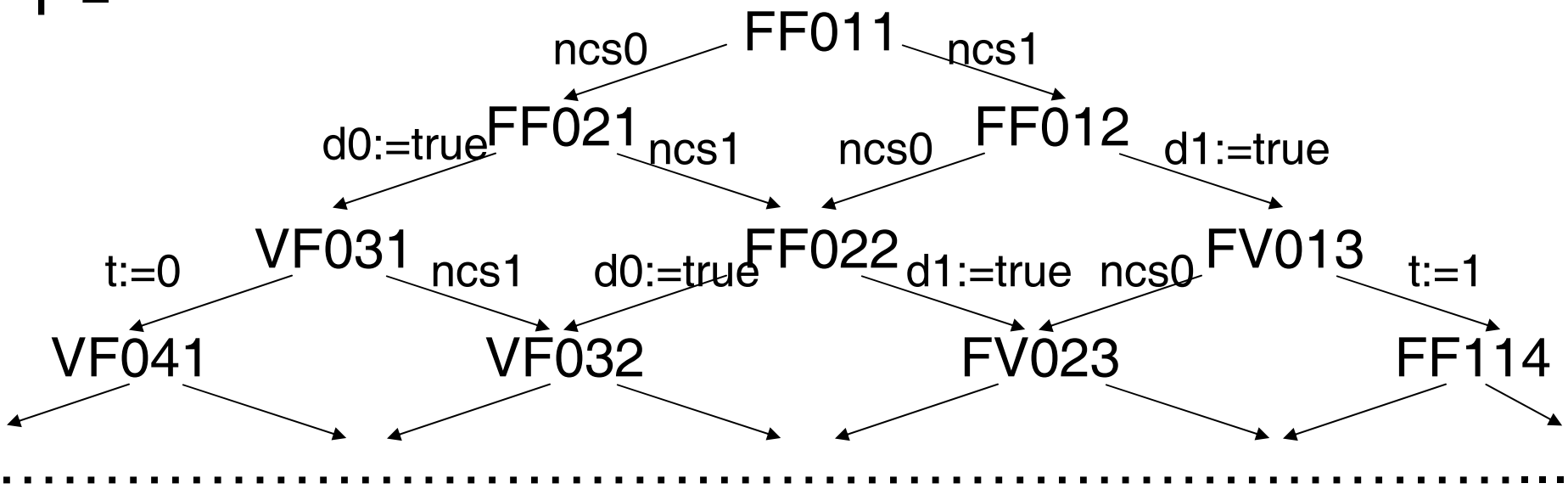
# Construction of the LTS

$S = \{ F,V \} \times \{ F,V \} \times \{ 0,1 \} \times \{ 1..7 \} \times \{ 1..7 \}$

$A = \{$ ncs0, ncs1, …, “d0:=true”, … $\}$

$s_0 = \langle F, F, 0, 1, 1 \rangle = FF011$

T =



# Remarks

- The LTS of Peterson's algorithm is finite:

$$| S | \cong 50 \leq 2 \times 2 \times 2 \times 7 \times 7 = 392$$

- In the presence of synchronizations, the number of reachable states is (much) smaller than the size of the cartesian product of the variable domains
- Some tools of CADP for LTS manipulation:
  - OCIS (step-by-step and guided simulation)
  - Executor (random exploration)
  - Exhibitor (search for regular sequences)
  - Terminator (search for deadlocks)

➔ can be used in conjunction with Exp.Open

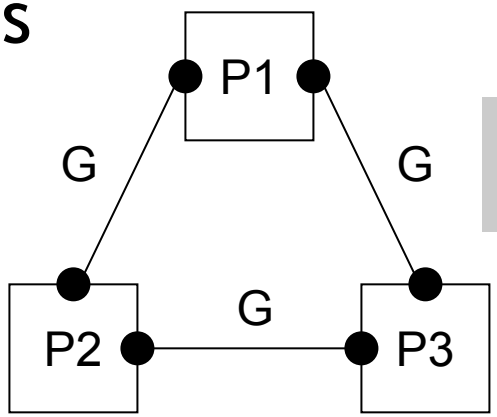
# Verification

- Once the LTS is generated, one can formulate and verify automatically the desired properties of the system
- For Peterson's algorithm:
  - Deadlock freedom: each state has at least one successor
  - Mutual exclusion: at most one process can be in the critical section at a given time
  - Liveness: no process can indefinitely overtake the other when accessing its critical section

[see the chapter on temporal logics]

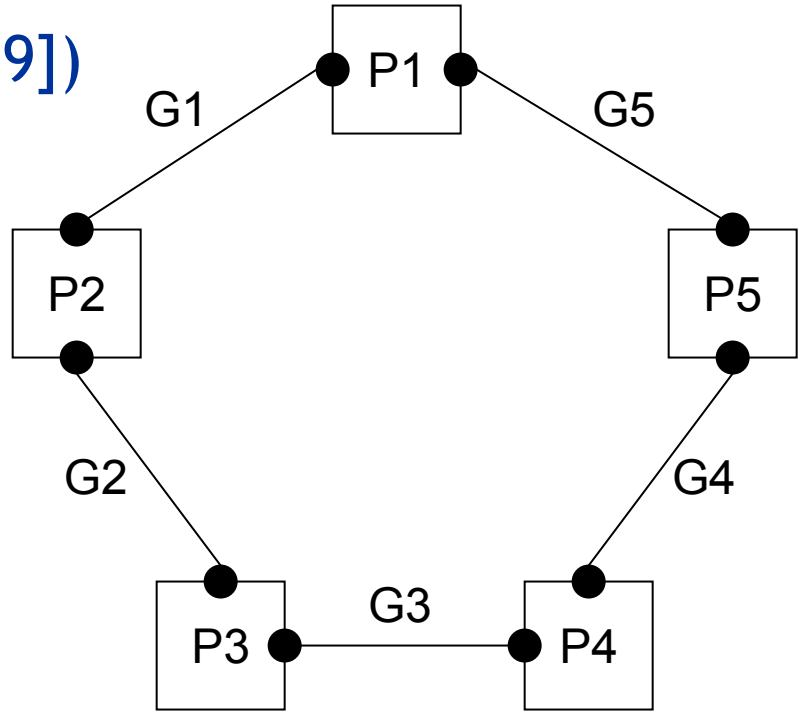
# Limitations of binary parallel composition

- Several ways of modeling a process network:
  - Absence of *canonical form*
  - Difficult to determine whether two composition expressions denote the same process network
  - Difficult to retrieve the process network from a composition expression
- The semantics of "|[$G_1$, ..., $G_n$]|" (rule $R_3$) does not prevent that other processes synchronize on $G_1$, ..., $G_n$ (*maximal cooperation*)
- Some networks cannot be modeled using "|[]|":




*binary synchro-nization on G*

# Example
## (ring network [Garavel-Sighireanu-99])




- Description using binary parallel composition:

$$(P_1 \;|[G_1]|\; P_2 \;|[G_2]|\; P_3 \;|[G_3]|\; P_4) \;|[G_4, G_5]|\; P_5$$

*the composition expression does not reflect the symmetry of the process network*

# General parallel composition
[Garavel-Sighireanu-99]

- "Graphical" parallel composition operator allowing the composition of several automata and their *m* among *n* synchronization:

**par** [ $g_1\#m_1$, …, $g_p\#m_p$ **in** ]
  $\underline{G}_1 \rightarrow B_1$
||  $\underline{G}_2 \rightarrow B_2$
. . .
||  $\underline{G}_n \rightarrow B_n$
**end par**

*gates with their associated synchronization degrees*

*automata (processes)*

*communication interfaces (gate lists)*

# General parallel composition
## (semantics – rules without synchronization degrees)

$$\frac{\exists\, a, i \,.\, B_i \xrightarrow{a} B_i' \wedge a \notin G_i \wedge \forall\, j \neq i \,.\, B_j' = B_j}{\textbf{par}\ G_1 \to B_1, \ldots, G_n \to B_n \xrightarrow{a} \textbf{par}\ G_1 \to B_1', \ldots, G_n \to B_n'} \quad (GR1)$$

*mandatory interleaved execution of non-synchronized actions*

$$\frac{\exists\, a. \forall\, i \,.\, \text{if } a \in G_i \text{ then } B_i \xrightarrow{a} B_i' \text{ else } B_j' = B_j}{\textbf{par}\ G_1 \to B_1, \ldots, G_n \to B_n \xrightarrow{a} \textbf{par}\ G_1 \to B_1', \ldots, G_n \to B_n'} \quad (GR2)$$

*execution in maximal cooperation of synchronized actions*

# Example (1/3)

- Process network unexpressible using "|[]|":

- Description using general parallel composition:

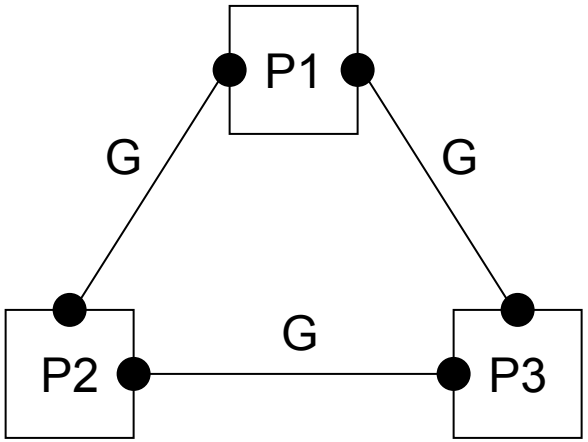
**par** G#2 **in**

G → $P_1$

|| G → $P_2$

|| G → $P_3$

**end par**




*maximal cooperation avoided by means of synchronization degrees*

# Example (2/3)
## (ring network [Garavel-Sighireanu-99])

- Description using general parallel composition:

**par**

$G_1, G_5 \rightarrow P_1$

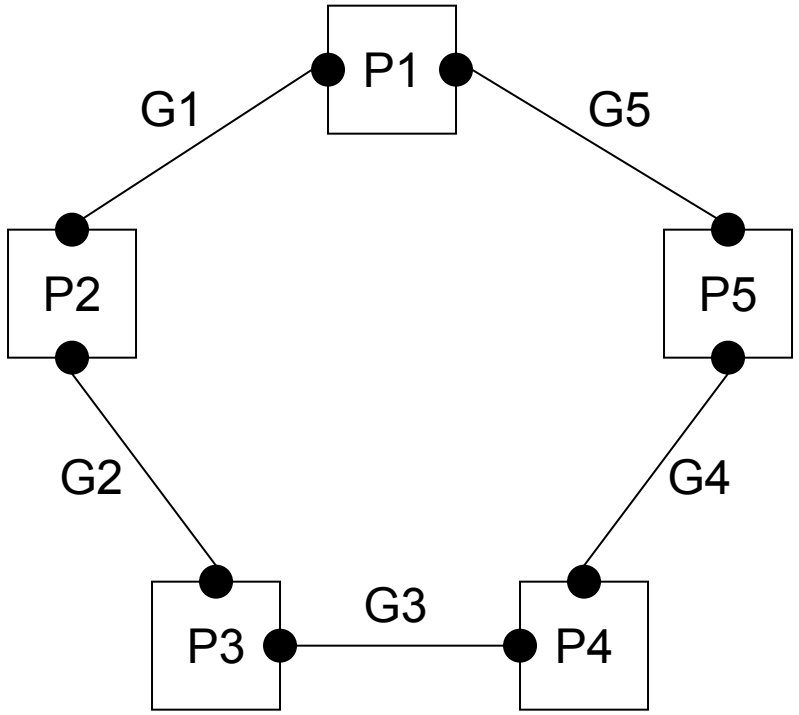
|| $G_2, G_1 \rightarrow P_2$

|| $G_3, G_2 \rightarrow P_3$

|| $G_4, G_3 \rightarrow P_4$

|| $G_5, G_4 \rightarrow P_5$

**end par**




*the symmetry of the process network is also present in the composition expression*

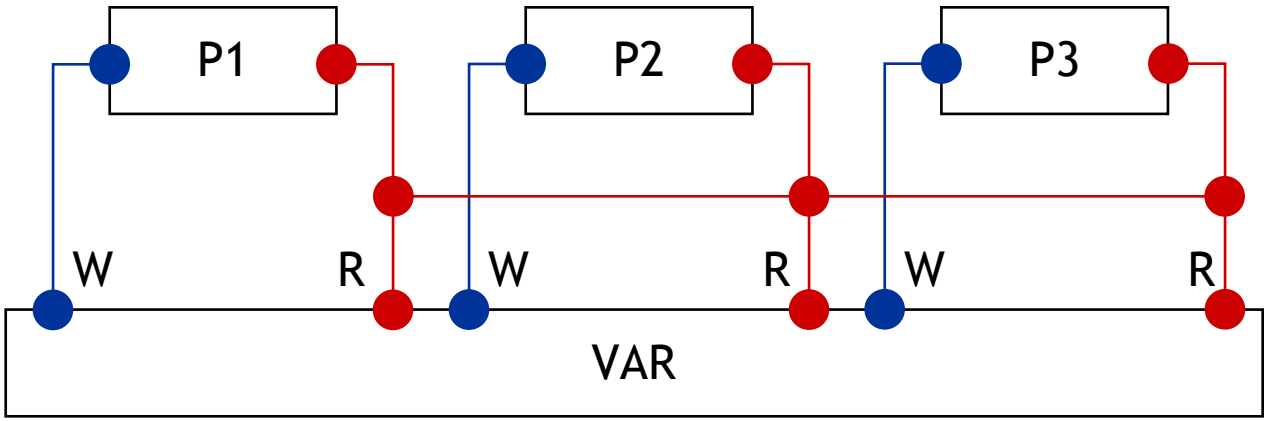
# Example (3/3)

- Definition of "|[]|" in terms of "**par**":

  $B_1$ |[$G_1$, ..., $G_n$]| $B_2$ = **par** $G_1$, ..., $G_n$ → $B_1$
  || $G_1$, ..., $G_n$ → $B_2$
  **end par**

- CREW (Concurrent Read / Exclusive Write):

  **par** W#2 **in**
  R, W → $P_1$
  || R, W → $P_2$
  || R, W → $P_3$
  || R, W → VAR
  **end par**



# Parallel composition using synchronization vectors

- Primitive form of n-ary parallel composition
- Proposed in various networks of automata: MEC [Arnold-Nivat], FC2 [deSimone-Bouali-Madelaine]
- Synchronizations are made explicit by means of *synchronization vectors*
- Syntax in the EXP language [Lang-05]:

**par** $V_1, \ldots, V_m$ **in**

$B_1 \,||\, \ldots \,||\, B_n$

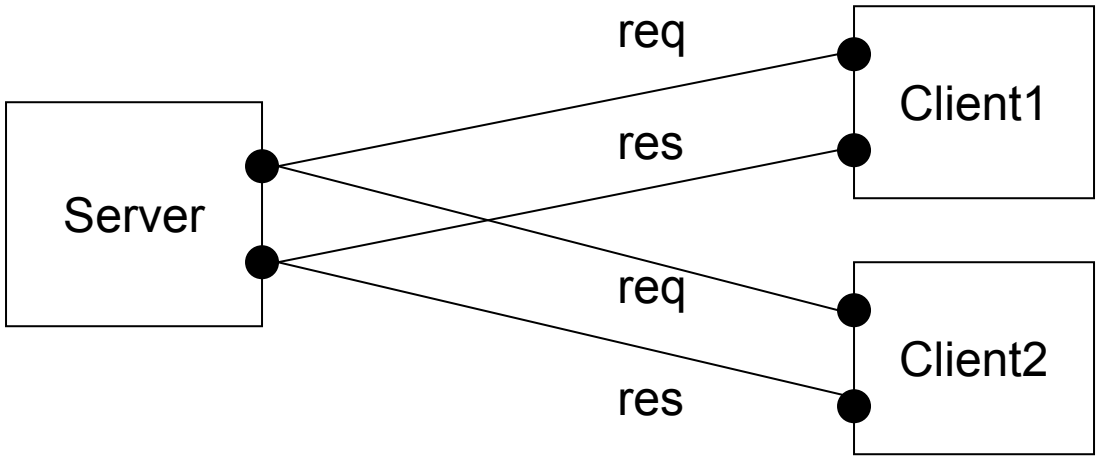
**end par**

$V ::= (G_1 \mid \_) * \ldots * (G_n \mid \_) \rightarrow G_0$

*synchronization vectors*

*wildcard*

# Example
## (client-server with gate multiplexing)




*binary synchronization on gates* req *and* res

- Description using synchronization vectors:

**par** req * _ * req → req, rep * _ * rep → rep,
_ * req * req → req, _ * rep * rep → rep
**in**
$Client_1$ || $Client_2$ || Server
**end par**

# Behavioural equivalence

- Useful for determining whether two LTSs denote the same behaviour
- Allows to:
  - Understand the semantics of languages (communicating automata, process algebras) having LTS models
  - Define and assess translations between languages
  - Refine specifications whilst preserving the equivalence of their corresponding LTSs
  - Replace certain system components by other, equivalent ones (maintenance)
  - Exploit identities between behaviour expressions (e.g., $B_1 \,|[G]|\, B_2 = B_2 \,|[G]|\, B_1$) in analysis tools

# Equivalence relations between LTSs


- A large spectrum of equivalence relations proposed:
  - *Trace* equivalence (≅ language equivalence)
  - *Strong* bisimulation [Park-81]
  - *Weak* bisimulation [Milner-89]
  - *Branching* bisimulation [Bergstra-Klop-84]
  - Safety equivalence [Bouajjani-et-al-90]
  - ...

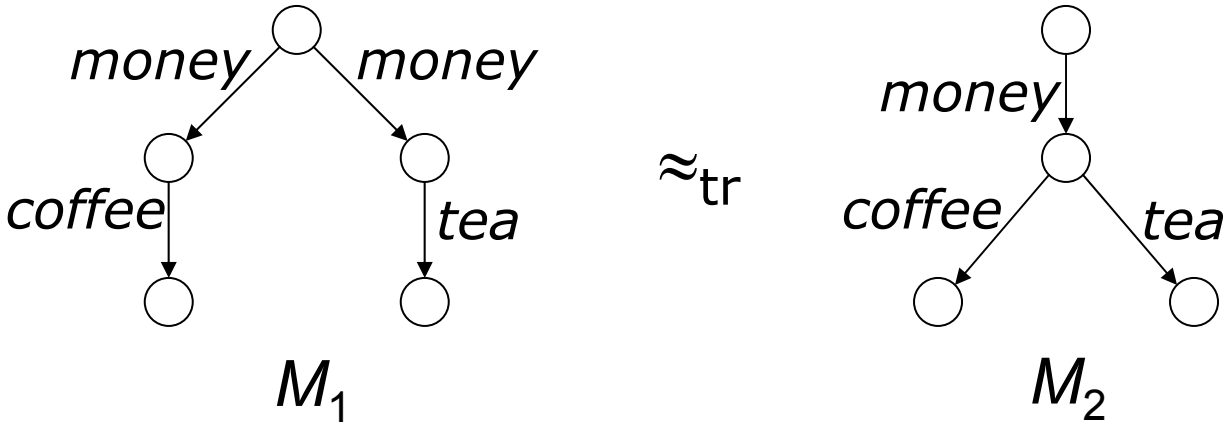
# Trace equivalence

- Trace: sequence of visible actions (e.g., $\sigma = req_1\ res_1\ req_2\ res_2$)
- Notations ($a$ = visible action):
  - $s\ =a=>$: there exists a transition sequence $s\ -i\rightarrow\ s_1\ -i\rightarrow\ s_2\ \ldots\ -a\rightarrow\ s_k$
  - $s\ =\sigma=>$: there exists a transition sequence $s\ =a_1=>\ s_1\ \ldots\ =a_n=>\ s_n$ such that $\sigma = a_1\ \ldots\ a_n$
- Two state are trace equivalents iff they are the source of the same traces:

$$s \approx_{tr} s' \quad \text{iff} \quad \forall \sigma\ .\ (s\ =\sigma=> \quad \text{iff} \quad s\ =\sigma=>)$$

# Example
## (coffee machine)

- The two LTSs below are trace equivalent:




Traces ($M_1$) = Traces ($M_2$) =
{ ε, *money*, *money coffee*, *money tea* }

➔ *have the two coffee machines the same behaviour w.r.t. a user?*

*$M_1$: risk of deadlock*

# Bisimulation

- Trace equivalence is not sufficiently precise to characterize the behaviour of a system w.r.t. its interaction with its environment

  ➔ *stronger relations (bisimulations) are necessary*

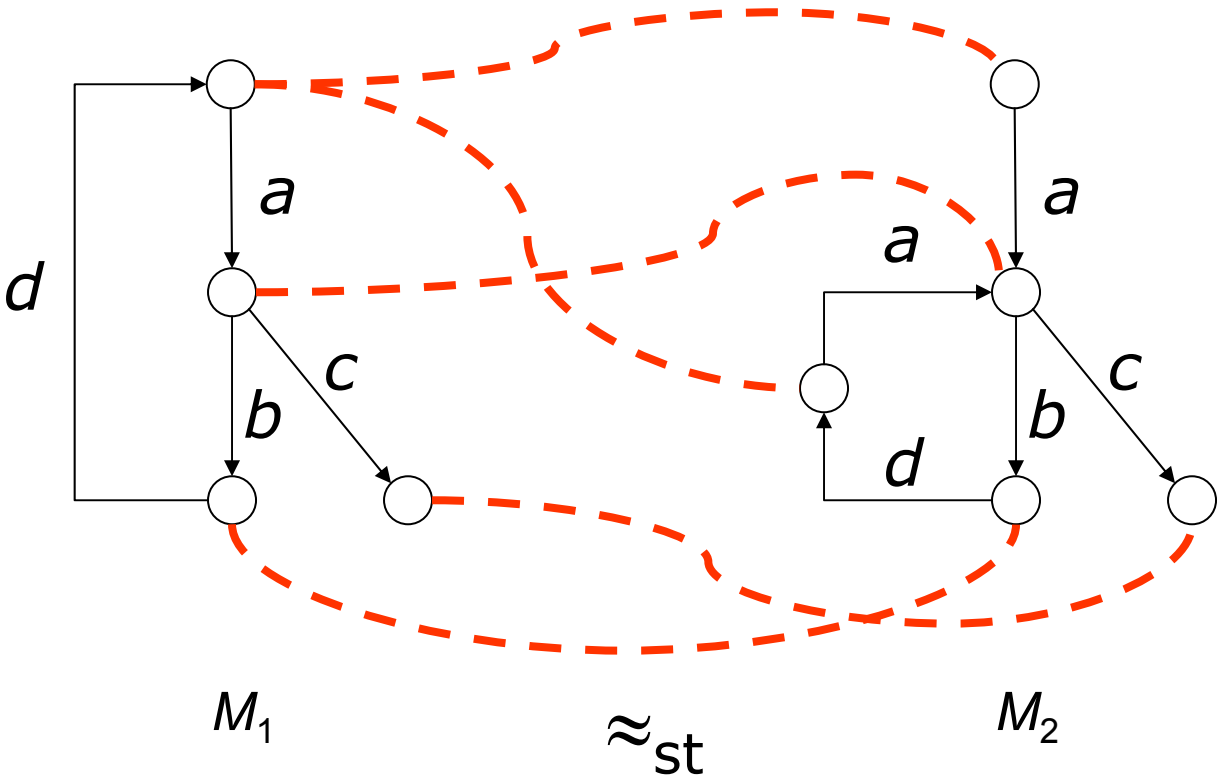
- Two states $s_1$ et $s_2$ are *bisimilar* iff they are the origin of the same behaviour (execution tree):

  $\forall\ s_1\text{-}a\rightarrow s_1'\ .\ \exists\ s_2\text{-}a\rightarrow s_2'\ \ .\ s_1' \approx s_2'$

  $\forall\ s_2\text{-}a\rightarrow s_2'\ .\ \exists\ s_1\text{-}a\rightarrow s_1'\ \ .\ s_2' \approx s_1'$

- Bisimulation is an equivalence relation (reflexive, symmetric, and transitive) on states
- Two LTSs are bisimilar iff $s_{01} \approx s_{02}$

# Strong bisimulation



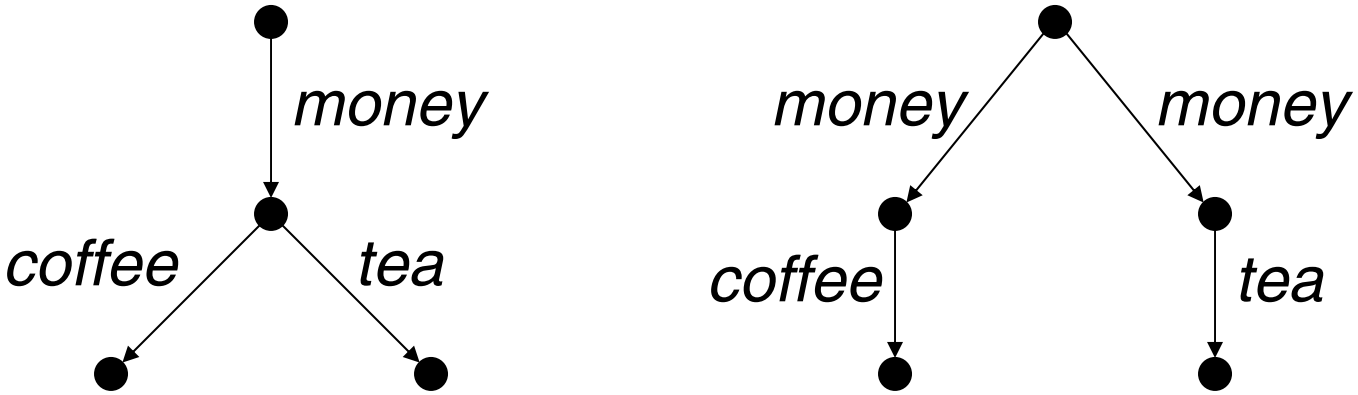

- Strong bisimulation: the largest bisimulation

  ➔ *to show that two LTSs are strongly bisimilar, it is sufficient to find a bisimulation between them*

# Is strong bisimulation sufficient?

- *Trace equivalence* ignores internal actions (*i*) and does not capture the branching of transitions

  ➔ *does not distinguish the LTSs below*



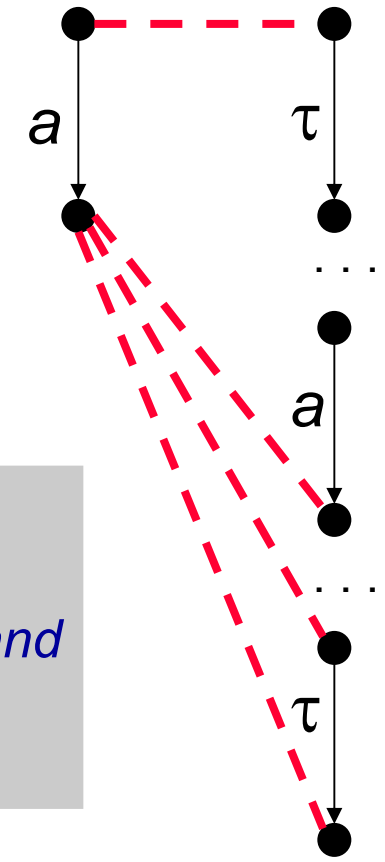

- *Strong bisimulation* captures the branching, but handles internal and visible actions in the same way

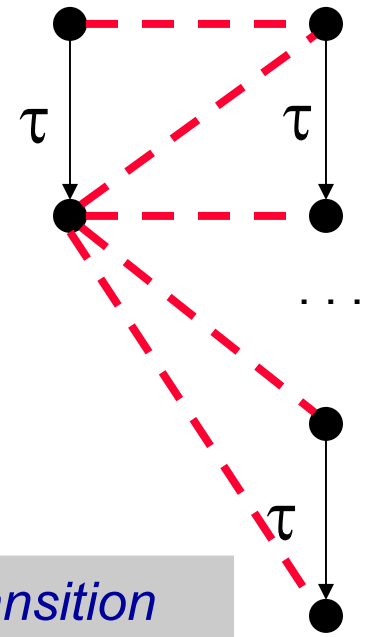
  ➔ *does not abstract away the internal behaviour*

# Weak bisimulation
(or *observational equivalence*)

- In practice, it is necessary to compare LTSs
  - By abstracting away internal actions
  - By distinguishing the branching
- *Weak bisimulation* [Milner-89]:

*every a-transition corresponds to an a-transition preceded and followed by 0 or more $\tau$-transitions*




*every $\tau$-transition corresponds to 0 or more $\tau$-transitions*

# Weak bisimulation
## (formal definition)

- Let $M_1 = \langle S_1, A, T_1, s_{01} \rangle$ and $M_2 = \langle S_2, A, T_2, s_{02} \rangle$
- A weak bisimulation is a relation $\approx \subseteq S_1 \times S_2$ such that $s_1 \approx s_2$ iff:

$$\forall\ s_1 \xrightarrow{a} s_1' \ .\ \exists\ s_2 \xrightarrow{\tau^*.a.\tau^*} s_2' \ .\ s_1' \text{ eq } s_2'$$

$$\forall\ s_1 \xrightarrow{\tau} s_1' \ .\ \exists\ s_2 \xrightarrow{\tau^*} s_2' \ .\ s_1' \text{ eq } s_2'$$

and

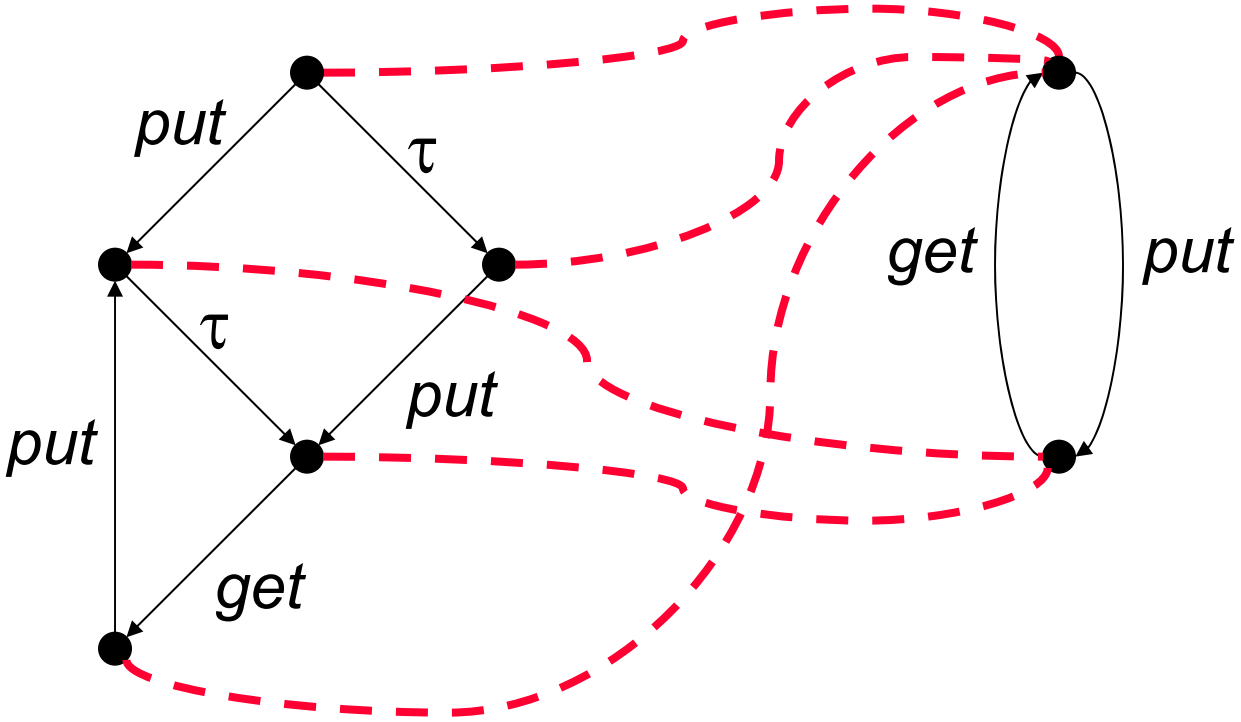
$$\forall\ s_2 \xrightarrow{a} s_2' \ .\ \exists\ s_1 \xrightarrow{\tau^*.a.\tau^*} s_1' \ .\ s_1' \text{ eq } s_2'$$

$$\forall\ s_2 \xrightarrow{\tau} s_2' \ .\ \exists\ s_1 \xrightarrow{\tau^*} s_1' \ .\ s_1' \text{ eq } s_2'$$

- $\approx_{obs}$ is the largest weak bisimulation
- $M_1 \approx_{obs} M_2$ iff $s_{01} \approx_{obs} s_{02}$

# Example

- To show that two LTSs are weakly bisimilar, it is sufficient to find a weak bisimulation between them



# Communicating automata
(summary)

- **Advantages:**
  - Simple model for describing concurrency
  - Powerful tools for manipulation
    - MEC (University of Bordeaux)
    - Auto/Autograph/FC2 (INRIA, Sophia-Antipolis)
    - CADP (INRIA, Grenoble)
  - Some industrial applications
- **Shortcomings:**
  - Limited expressiveness
    - No dynamic creation and destruction of automata
    - Impossible to express: A then (B || C) then D
    - No handling of data (each variable = an automaton), unacceptable for complex types (numbers, lists, structures, ...)
  - Maintenance difficult and error-prone (large automata)

# Process algebraic languages

- Basic notions
- Parallel composition and hiding
- Sequential composition and choice
- Value-passing and guards
- Process definition and instantiation

# Process algebras

- PAs: theoretical formalisms for describing and studying concurrency and communication
- Examples of PAs for asynchronous systems:
  - CCS (*Calculus of Communicating Systems*) [Milner-89]
  - CSP (*Communicating Sequential Processes*) [Hoare-85]
  - ACP (*Algebra of Communicating Processes*) [Bergstra-Klop-84]
- Basic idea of PAs:
  - Provide a small number of operators
  - Construct behaviours by freely combining operators (lego)
- Standardized specification languages:
  - LOTOS [ISO-1988], E-LOTOS [ISO-2001]

# LOTOS
## (Language Of Temporal Ordering Specification)

- International standard [ISO 8807] for the formal specification of telecommunication protocols and distributed systems

  **http://www.inrialpes.fr/vasy/cadp/tutorial**

- Enhanced LOTOS (E-LOTOS): revised standard [2001]
- LOTOS contains two "orthogonal" sublanguages:
  - *data* part (for data structures)
  - *process* part (for behaviours)
- Handling data is necessary for describing realistic systems. "Basic LOTOS" (the dataless fragment of LOTOS) is useful only for small examples.

# LOTOS – data part

- Based on algebraic abstract data types (ActOne):

```
type Natural is
   sorts Nat
   opns  0 : -> Nat
         succ : Nat -> Nat
         + : Nat, Nat -> Nat
   eqns forall M, N : Nat
     ofsort Nat
          0 + N = N;
          succ(M) + N = succ(M + N);
endtype
```

- Caesar.Adt compiler of CADP [Garavel-Turlier-92]
- ADTs tend to become cumbersome for complex data manipulations (removed in E-LOTOS).

# LOTOS - process part

- Combines the best features of the process algebras CCS [Milner-89] and CSP [Hoare-85]
- Terminal symbols (identifiers):
  - Variables: $X_1$, …, $X_n$
  - Gates: $G_1$, …, $G_n$
  - Processes: $P_1$, …, $P_n$
  - Sorts (≈ types): $S_1$, …, $S_n$
  - Functions: $F_1$, …, $F_n$
  - Comments: (* … *)
- Caesar compiler of CADP [Garavel-Sifakis-90]

# Value expressions and offers

- Value expressions: $V_1$, …, $V_n$

  $V$ ::= $X$

  | $F\ (V_1, \ldots, V_n)$

  | $V_1\ F\ V_2$

- Offers: $O_1$, …, $O_n$

  $O$ ::= ! $V$ — emission of a value $V$

  | ? $X$ : $S$ — reception of a value to be stored in a variable $X$ of sort $S$

# Behaviour expressions
## (Lots Of Terribly Obscure Symbols :-)

- Behaviours: $B_1$, …, $B_n$

| | | |
|---|---|---|
| $B$ ::= | **stop** | inaction |
| \| | $G_0\, O_1 \ldots O_n\, [\ V\ ]\ ;\ B_0$ | action prefix |
| \| | $B_1$ [] $B_2$ | choice |
| \| | $B_1$ \|[ $G_1$, …, $G_n$ ]\| $B_2$ | parallel with synchronization on $G_1$, …, $G_n$ |
| \| | $B_1$ \|\|\| $B_2$ | interleaving |
| \| | **hide** $G_1$, …, $G_n$ **in** $B_0$ | hiding |
| \| | [ $V$ ] -> $B_0$ | guard |
| \| | **let** $X$ : $S$ = $V$ **in** $B_0$ | variable definition |
| \| | **choice** $X$ : $S$ [] $B_0$ | choice over values |
| \| | P [ $G_1$, …, $G_n$ ] ($V_1$, …, $V_n$) | process call |

# Process definitions

**process** $P$ [ $G_1$, …, $G_n$ ] ($X_1$:$S_1$, …, $X_n$:$S_n$) :=

$B$

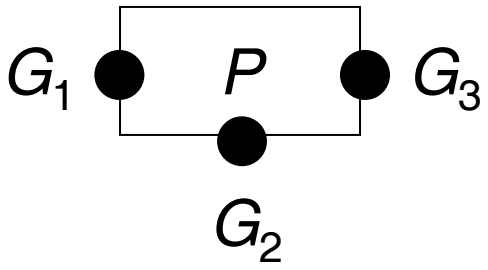
**endproc**

where:

- $P$ = process name
- $G_1$, …, $G_n$ = formal *gate* parameters of P
- $X_1$, …, $X_n$ = formal *value* parameters of $P$, of sorts $S_1$, …, $S_n$
- $B$ = body (behaviour) of $P$

# Remarks

- LOTOS process: “black box” equipped with communication points (gates) with the outside




**process** $P$ $[G_1, G_2, G_3]$ (...) :=

...

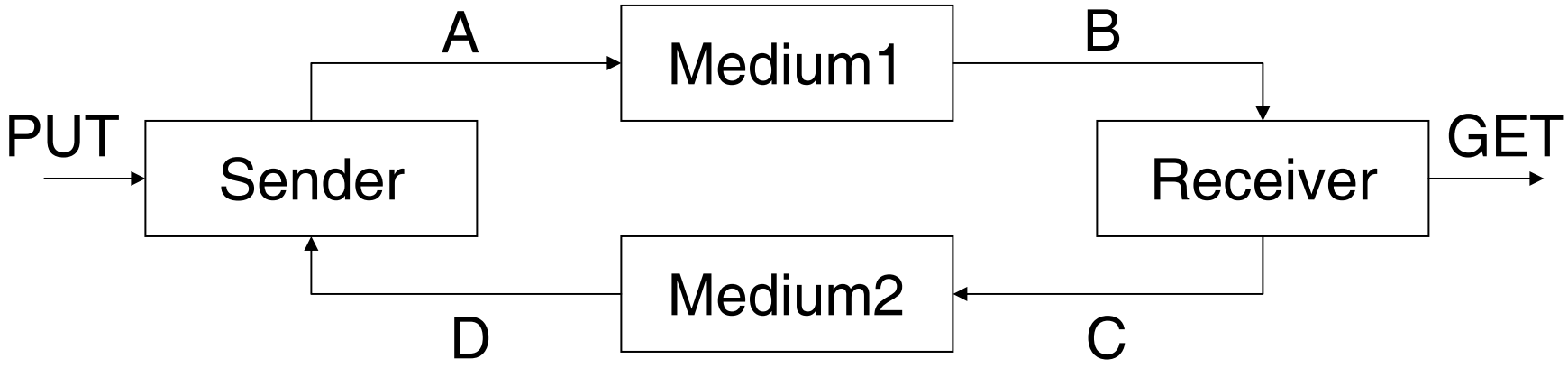
**endproc**

- Each process has its own local (private) variables, which are not accessible from the outside

  ➔ *communication by rendezvous and not by shared variables*

- Parallel composition and encapsulation of boxes: described using the |[...]|, |||, and **hide** operators

# Example



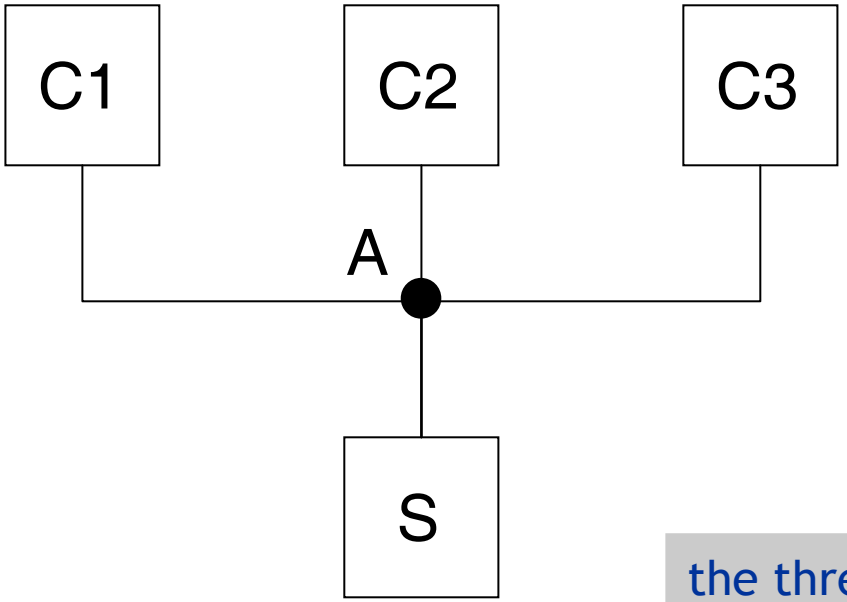

(Sender [PUT, A, D] ||| Receiver [GET, B, C])
|[A, B, C, D]|
(Medium1 [A, B] ||| Medium2 [C, D])

or

(Sender [PUT, A, D] |[A]| Medium1 [A, B])
|[B, D]|
(Receiver [GET, B, C] |[C]| Medium2 [C, D])

# Multiple rendezvous

- LOTOS parallel operators allow to specify the synchronization of $n \geq 2$ processes on the same gate



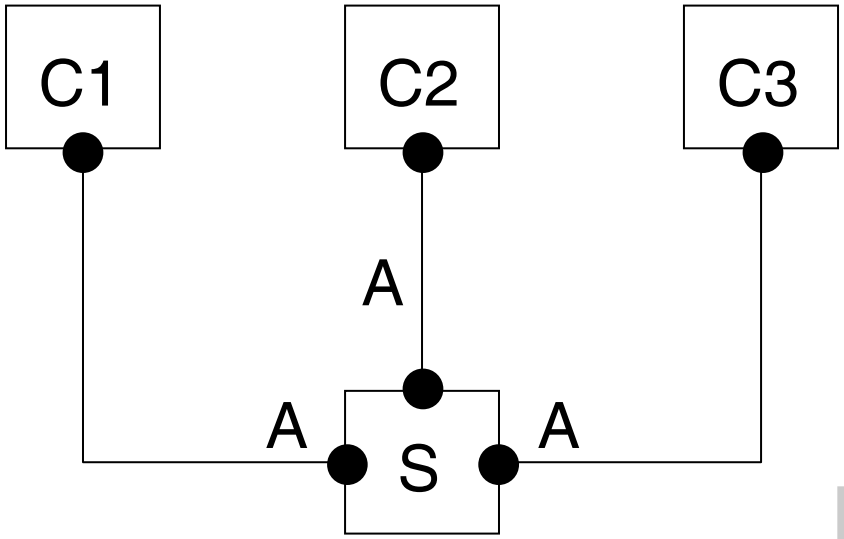

Example (client-server):

```
C1 [A] |[A]| C2 [A] |[A]| C3 [A]
  |[A]|
S [A]
```

the three client processes synchronize with the server on gate A (4-way rendezvous)

# Binary rendezvous

- The ||| operator allows to specify binary rendezvous (2 among *n*) on the same gate




Example (client-server):

```
(C1 [A] ||| C2 [A] ||| C3 [A])
|[A]|
S [A]
```

the three client processes are competing to access the server on gate A but only one can get access at a given moment

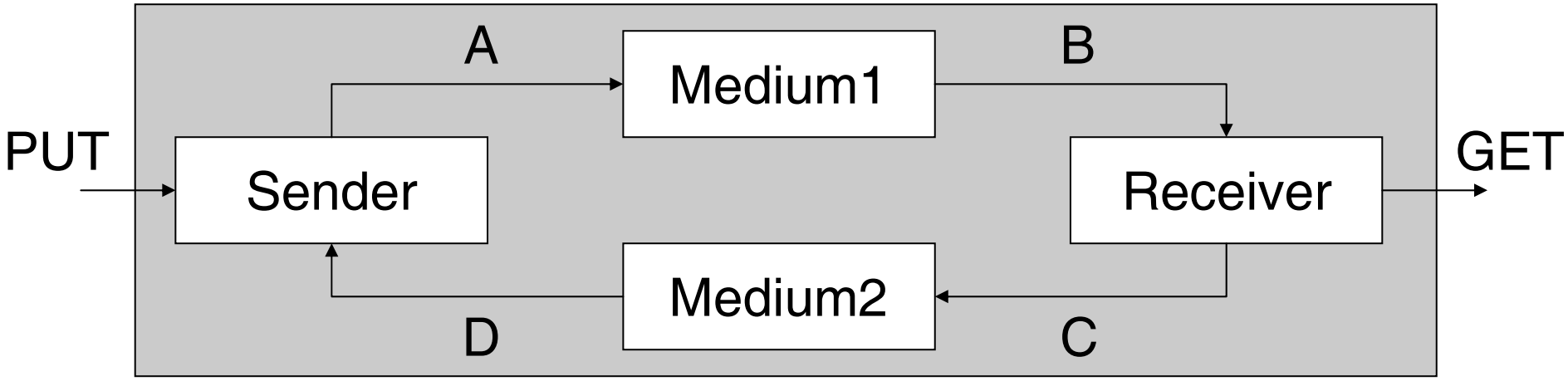
# Abstraction
(hiding)

- In LOTOS, when a synchronization takes place on a gate G between two processes, another one can also synchronize on G (*maximal cooperation*)
- If this is undesirable, it can be forbidden by hiding the gate (renaming it into ***i***) using the **hide** operator:

  **hide** $G_1$, …, $G_n$ **in** $B$

  which means that all actions performed by $B$ on gates $G_1$, …, $G_n$ are hidden
- The gates $G_1$, …, $G_n$ are "abstracted away" (hidden from the outside world)

# Example




```
process Network [PUT, GET] :=
    hide A, B, C, D in
      (Sender [PUT, A, D] ||| Receiver [GET, B, C])
      |[A, B, C, D]|
      (Medium1 [A, B] ||| Medium2 [C, D])
endproc
```

# Operational semantics

- Notations:
  - *G*: gate list (or set)
  - *L*: action (transition label), of the form

    $G\ V_1, \ldots, V_n$

    where *G* is a gate and $V_1, \ldots, V_n$ is the list of values exchanged on *G* during the rendezvous
  - *gate* (*L*) = *G*
  - *B* [ *v* / *X* ]: syntactic substitution of all free occurrences of *X* inside *B* by a value *v* (having the same sort as *X*)
  - *V* [ *v* / *X* ]: idem, substitution of *X* by *v* in *V*

# Semantics of "|[...]|"

$$\frac{B_1 \rightarrow_L B_1' \wedge gate\ (L) \notin \underline{G}}{B_1\ |[\ \underline{G}\ ]|\ B_2 \rightarrow_L B_1'\ |[\ \underline{G}\ ]|\ B_2}$$

$B_1$ evolves

$$\frac{B_2 \rightarrow_L B_2' \wedge gate\ (L) \notin \underline{G}}{B_1\ |[\ \underline{G}\ ]|\ B_2 \rightarrow_L B_1\ |[\ \underline{G}\ ]|\ B_2'}$$

$B_2$ evolves

$$\frac{B_1 \rightarrow_L B_1' \wedge B_2 \rightarrow_L B_2' \wedge gate\ (L) \in \underline{G}}{B_1\ |[\ \underline{G}\ ]|\ B_2 \rightarrow_L B_1'\ |[\ \underline{G}\ ]|\ B_2'}$$

$B_1$ and $B_2$ evolve

- Gates have no direction of communication

# Semantics of "hide"

$$\frac{B \rightarrow_L B' \wedge gate\ (L) \notin \underline{G}}{\textbf{hide } \underline{G} \textbf{ in } B \rightarrow_L \textbf{hide } \underline{G} \textbf{ in } B'}$$

normal gate

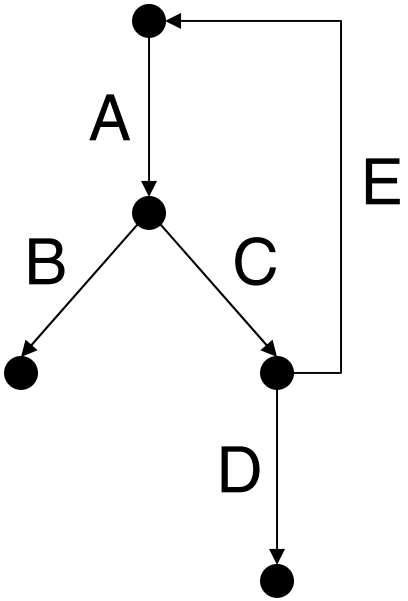
$$\frac{B \rightarrow_L B' \wedge gate\ (L) \in \underline{G}}{\textbf{hide } \underline{G} \textbf{ in } B \rightarrow_{\mathbf{i}} \textbf{hide } \underline{G} \textbf{ in } B'}$$

hidden gate

- In LOTOS, **i** is a keyword: use with care

# Sequential behaviours

- LOTOS allows to encode sequential automata by means of the choice (“[]”) and sequence operators (“;” and “**stop**”), and recursive processes




```
process P [A, B, C, D, E] : noexit :=
    A; (
        B; stop
        []
        C; (
            D ; stop
            []
            E ; P [A, B, C, D, E]
        )
    )
endproc
```

# Remarks

- The description of automata in LOTOS is not far from regular expressions (operators “.”, “|”, “*”), except that:
  - The “;” operator of LOTOS is *asymmetric* (≠ from “.”)

    $G\ O_1 \ldots O_n\ ;\ B$ but not $B_1\ ;\ B_2$
  - There is no iteration operator “*”, one must use a recursive process call instead

- LOTOS allows to describe automata with data values (≈ functions in sequential languages) by using processes with value parameters

# Semantics of “stop”

- The “**stop**” operator (inaction) has no associated semantic rule, because no transition can be derived from it

- A call of a “pathological” recursive process like

  **process** P [A] : **noexit** :=

  P [A]

  **endproc**

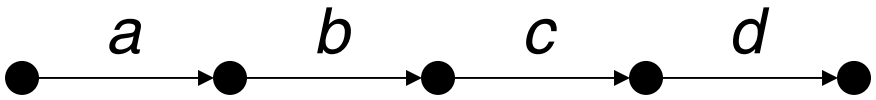
  has a behaviour equivalent to **stop** (unguarded recursion)

# Prefix operator (“;”)

- Allows to describe:
  - Sequential composition of actions
  - Communication (emission / reception) of data values

- Simplest variant: actions on gates, without value-passing (basic LOTOS)

*a* ; *b* ; *c* ; *d* ; **stop**



# Semantics of ";"

Case 1: action without reception offers (?*X*:*S*)

$$\frac{(\forall 1 \leq i \leq n \;.\; O_i \equiv !\; V_i\;) \wedge V = \text{true}}{G\; O_1 \ldots O_n\; [\; V\;]\; ;\; B \rightarrow_{G\; V1 \ldots Vn} B}$$

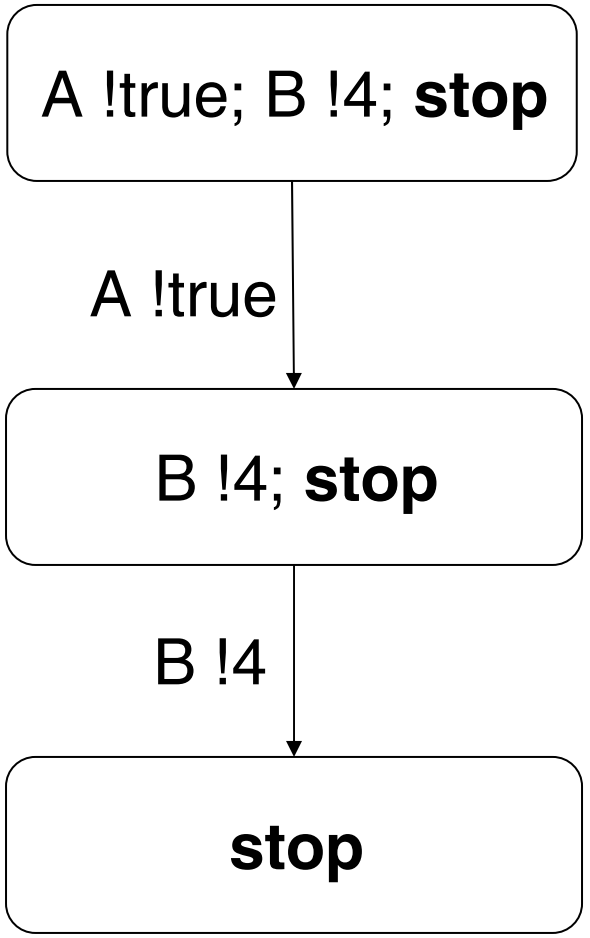
- The boolean guard and the offers are optional
- If the guard *V* is false, the rendezvous does not happen (deadlock):

$$G\; O_1 \ldots O_n\; [\; V\;]\; ;\; B \;\approx\; \textbf{stop}$$

# Example (1/2)

Sequential composition:

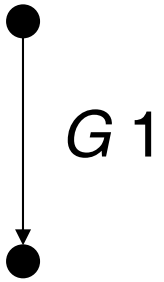
A !true; B !4; **stop**



# Example (2/2)

- Synchronization by *value matching*: two processes send to each other the same values on a gate

$G$ !1; $B_1$ |[ $G$ ]| $G$ !1; $B_2$ — RdV OK




$G$ !1; $B_1$ |[ $G$ ]| $G$ !2; $B_2$ — deadlock (different values)

$G$ !1; $B_1$ |[ $G$ ]| $G$ !true; $B_2$ — deadlock (different types)

# Semantics of ";"

Case 2: action containing reception offer(s) (?*X*:*S*)

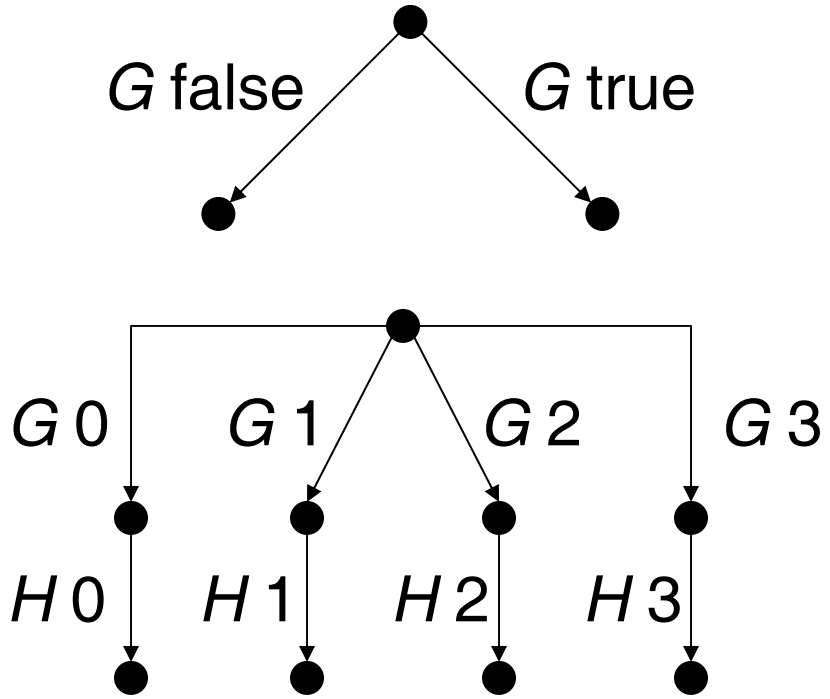
$$\frac{(v \in S) \wedge (V\,[\,v\,/\,X\,] = \text{true})}{G\ ?X{:}S\,[\,V\,]\ ;\ B \rightarrow_{G\,v} B\,[\,v\,/\,X\,]}$$

- The variables defined in the offers ?X:S are visible in the boolean guard *V* and inside *B*
- An action can freely mix emission and reception offers

# Example (1/3)

*G* ?*X*:Bool;
  **stop**

*G* ?*X*:Nat [*X* < 4];
  H ! X;
    **stop**



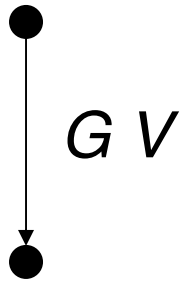

- The semantics handles the reception by branching on all possible values that can be received

# Example (2/3)

- Emission of a value = guarded reception:

  $G\ !V \equiv G\ ?X{:}S\ [\ X = V\ ]$

  where $S = type\ (V\ )$

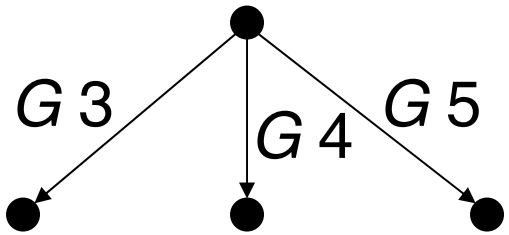



- Synchronization by *value generation*: two processes receive values of the same type on a gate

  $G\ ?n_1{:}Nat\ [\ n_1 <= 5\ ];\ B_1$

  $|[\ G\ ]|$

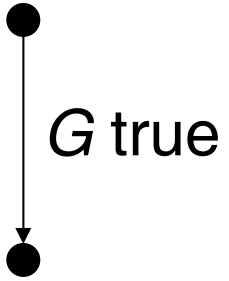
  $G\ ?n_2{:}Nat\ [\ n_2 > 2];\ B_2$



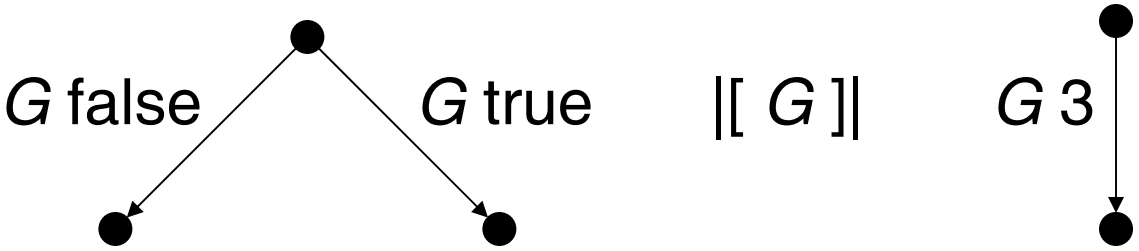
# Example (3/3)

- Synchronization by *value-passing*:

*G* ?*X*:Bool ; **stop** |[ *G* ]| *G* !true ; **stop**




*G* ?*X*:Bool ; **stop** |[ *G* ]| *G* !3 ; **stop**




deadlock: the semantics of the "|[...]|" operator requires that the two labels be identical (same type for the emitted value and the reception offer)

# Rendezvous
(summary)

- General form:

$$G\ O_1 \ldots O_m\ [V_1];\ B_1 \quad |[\ \underline{G}\ ]| \quad G'\ O_1' \ldots O_n'[V_2];\ B_2$$

- Conditions for the rendezvous:
  - $G = G'$ and $G \in \underline{G}$
  - $m = n$
  - $V_1$ and $V_2$ are true in the context of $O_1, \ldots, O_n'$
  - $\forall 1 \leq i \leq n.\ type\ (O_i) = type\ (O_i')$
  - $\forall 1 \leq i \leq n.\ prop\ (O_i) \cap prop\ (O_i') \neq \emptyset$

  where $prop(O)$ = set of values accepted by offer $O$
  - $prop\ (!V\ ) = \{\ V\ \}$
  - $prop\ (?X{:}S) = S$

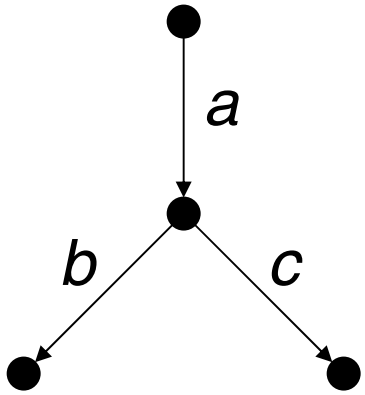
# Choice operator (“[]”)

- ”[]”: notation inherited from the programs with guarded commands [Dijkstra]
- Allows to specify the choice between several alternatives:

  ( $B_1$ [] $B_2$ [] $B_3$ )

  can execute either $B_1$, or $B_2$, or $B_3$
- Example:

  *a* ;
  (*b* ; **stop**
  []
  *c* ; **stop**)



# Semantics of "[]"

$$\frac{B_1 \rightarrow_L B_1'}{B_1 \; [] \; B_2 \rightarrow_L B_1'}$$

execution of $B_1$

$$\frac{B_2 \rightarrow_L B_2'}{B_1 \; [] \; B_2 \rightarrow_L B_2'}$$

execution of $B_2$

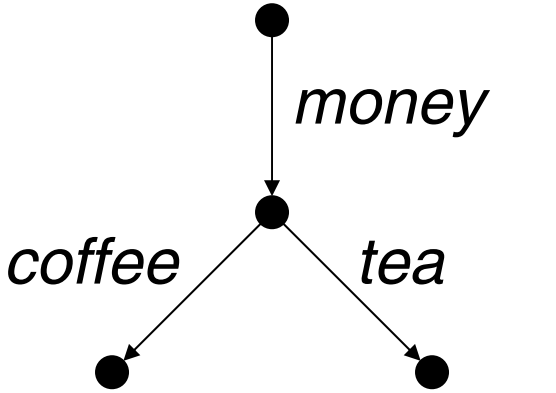
- After the choice, one of the two behaviours disappears (the execution was engaged on a branch of the choice and the other one is abandoned)

# Internal / external choice

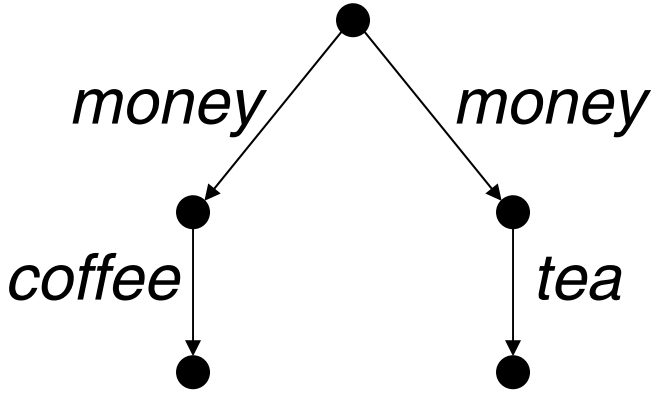
$(G_1 \; ; \; B_1 \quad [] \quad G_2 \; ; \; B_2 \;)$

- External choice: the environment can decide which branch will be executed
- Internal choice: the program decides

- Example (coffee machine):




external choice (user)

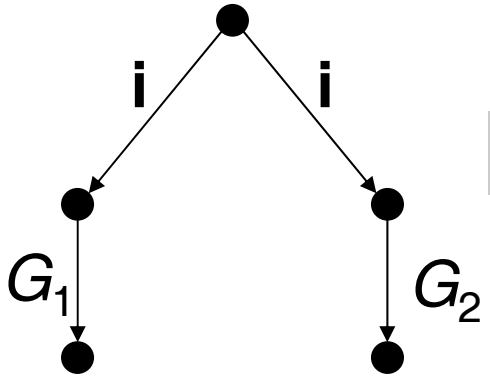



internal choice (machine)

# Internal action ("i")

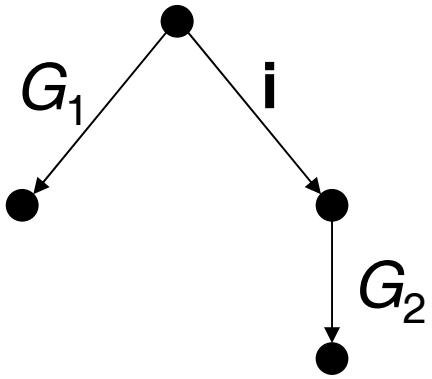
- In LOTOS, the special gate **i** denotes an internal event on which the environment cannot act:

**(i ; $G_1$ ; stop**
**[]**
**i ; $G_2$ ; stop)**




*internal choice*

**($G_1$ ; stop**
**[]**
**i ; $G_2$ ; stop)**




*still internal choice*

# Guard operator ("[...] ->")

- LOTOS does not possess an "if-then-else" construct
- *Guards* (boolean conditions) can be used instead
- Informal semantics:

  [ *V* ] -> *B* ≈ **if *V* then *B* else stop**

- Frequent usage in conjunction with "[]":

```
READ ?m,n:Nat ;
( [ m >= n ] -> PRINT !m; stop
  []
  [ m < n ]  -> PRINT !n; stop )
```

*emission of max (m,n) on gate PRINT*

# Semantics of "[…] ->"

$$\frac{(V = \text{true}) \wedge B \rightarrow_L B'}{[\ V\ ] \text{ -> } B \rightarrow_L B'}$$

- If the boolean expression *V* evaluates to false, no semantic rule applies (deadlock):

  [ false ] -> *B* ≈ **stop**

# Examples

- "if-then-else":

  [ $V$ ] -> $B_1$
  []
  [ not ($V$ ) ] -> $B_2$

  "case":

  [ $X < 0$ ] -> $B_1$
  []
  [ $X = 0$ ] -> $B_2$
  []
  [ $X > 0$ ] -> $B_3$

- Beware of overlapping guards:

  [ $X \leq 0$ ] -> $B_1$
  []
  [ $X \geq 0$ ] -> $B_2$

*if X = 0 then this is equivalent to an unguarded choice B1 [] B2*

# Operator “let”

- LOTOS allows to define variables for storing the results of expressions
- Variable definition:

  **let** *X*:*S* = *V* **in** *B*

  declares variable *X* and initializes it with the value of *V*. *X* is visible in *B*.
- *Write-once* variables (no multiple assignments):

  **let** *X*:*Bool* = *true* **in** *G* !*X* ; (* first *X* *)

  **let** *X*:*Bool* = *false* **in** *G* !*X* ; (* second *X* *)

  **stop**

# Semantics of “let”

$$\frac{B\,[\,V\,/\,X\,] \rightarrow_L B'}{\textbf{let } X{:}S = V \textbf{ in } B \rightarrow_L B'}$$

- Example:

  **let** *X*:NatList = cons (0, nil) **in**

  *G* !*X*;

  *H* !cons (1, *X* );

  **stop**

# Remarks

LOTOS is a *functional* language:

- No uninitialized variable (forbidden by the syntax)
- No assignment operator (":="), the value of a variable does not change after its initialization
- No "global" or "shared" variables between functions or processes
- Each process has its own local variables
- Communication by rendezvous only
- No side-effects

# Operator “choice”

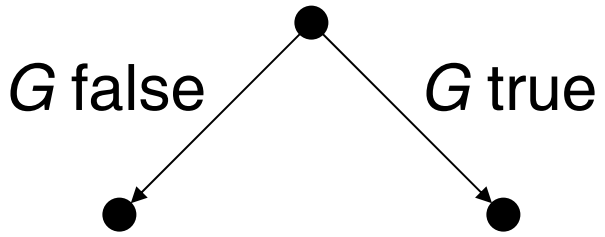
- Operator “**choice**”: similar to “**let**”, except that variable *X* takes a nondeterministic value in the domain of its sort *S*
- Semantics:

$$\frac{(v \in S) \wedge B\,[\,v\,/\,X\,] \rightarrow_L B'}{\textbf{choice}\ X{:}S\ []\ B \rightarrow_L B'}$$

- Example:

**choice** *X*:Bool []

*G* !*X*; **stop**



# Examples

- Reception of a value = particular case of “**choice**”:

  *G* ?*X*:*S* ; *B* = **choice** *X*:*S* [] *B*

- Iteration over the values of an enumerated type:

  **choice** *A*:*Addr* []

  *SEND* !*m* !*A* ; **stop**

- Generation of a random value:

  **choice** *rand*:*Nat* []

  [ *rand* <= 10 ] -> *PRINT* !*rand* ; **stop**

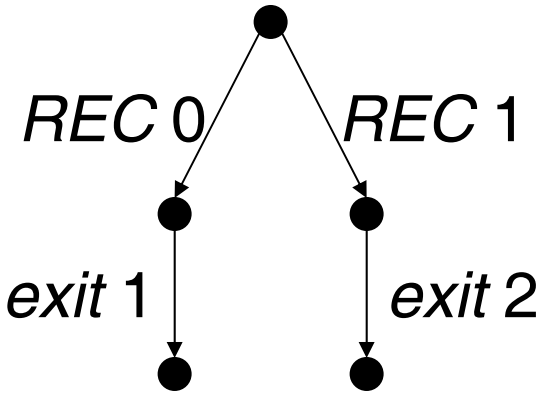
# Operator “exit”

- LOTOS allows to express *normal termination* of a behaviour, possibly with the return of one or several values:

  **exit** ( $V_1$, …, $V_n$ )

  denotes a behaviour that terminates and produces the values $V_1$, …, $V_n$
- Example:

*REC* ?*x*:Nat [ *x* < 2 ] ;
  **exit** (*x* + 1)



# Semantics of “exit”

$$\frac{\text{true}}{\textbf{exit}\ (\ V_1,\ \ldots,\ V_n\ ) \rightarrow_{exit\ V1\ \ldots\ Vn}\ \textbf{stop}}$$

- *exit* = special gate, synchronized by the “|[…]|” operator (see later)
- The values $V_1$, …, $V_n$ are optional (“**exit**” means normal termination without producing any value)

# Operator “>>”

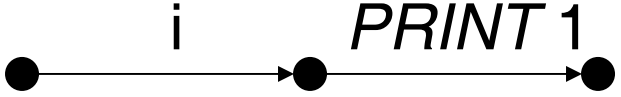
- LOTOS allows to express the sequential composition between a behaviour $B_1$ that terminates and a behaviour $B_2$ that begins:

  $B_1$ >> **accept** $X_1$:$S_1$,..., $X_n$:$S_n$ **in** $B_2$

  means that when $B_1$ terminates by producing values $V_1$,..., $V_n$, the execution continues with $B_2$ in which $X_1$,..., $X_n$ are replaced by the values $V_1$,..., $V_n$

- Example:

```
exit (1) >> accept n:Nat in
    PRINT !n ; stop
```



# Semantics of “>>”

$$\frac{(B_1 \rightarrow_L B_1' ) \wedge (gate\ (L) \neq exit\ )}{(B_1 \gg \textbf{accept}\ \underline{X}:\underline{S}\ \textbf{in}\ B_2) \rightarrow_L (B_1' \gg \textbf{accept}\ \underline{X}:\underline{S}\ \textbf{in}\ B_2)}$$

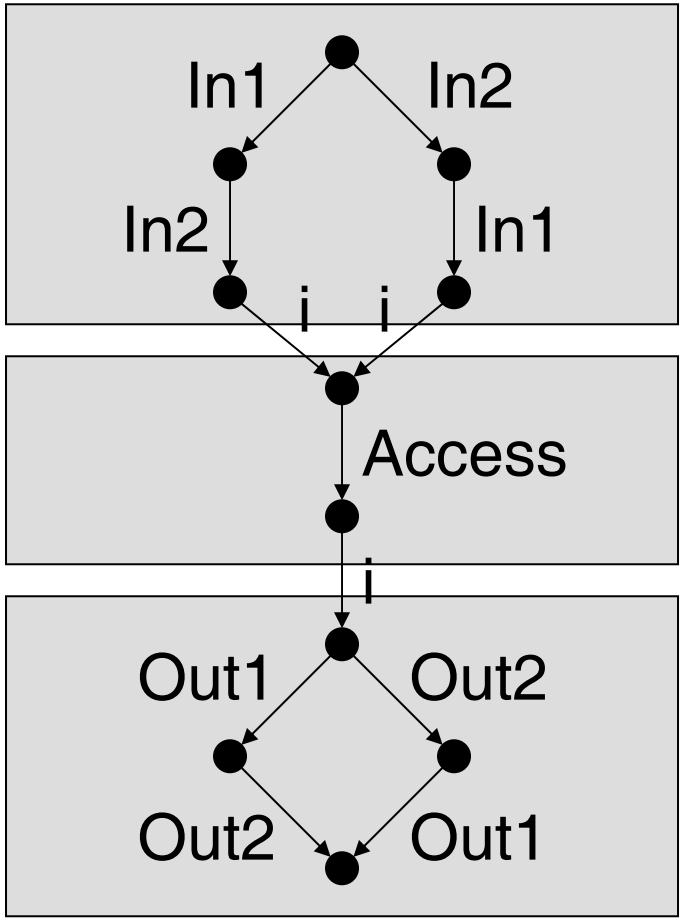
$$\frac{B_1 \rightarrow_{exit\ \underline{V}} B_1'}{(B_1 \gg \textbf{accept}\ \underline{X}:\underline{S}\ \textbf{in}\ B_2) \rightarrow_{\mathbf{i}} B_2\ [\ \underline{V}\ /\ \underline{X}\ ]}$$

- The $\underline{V}$ values must belong pairwise to the $\underline{S}$ sorts
- The *exit* gate is hidden (renamed into **i**) when sequential composition takes place
- The “>>” operator is also called *enabling* ($B_2$’s execution is made possible by $B_1$’s termination)

# Example (1/4)

- Sequential composition without value-passing:

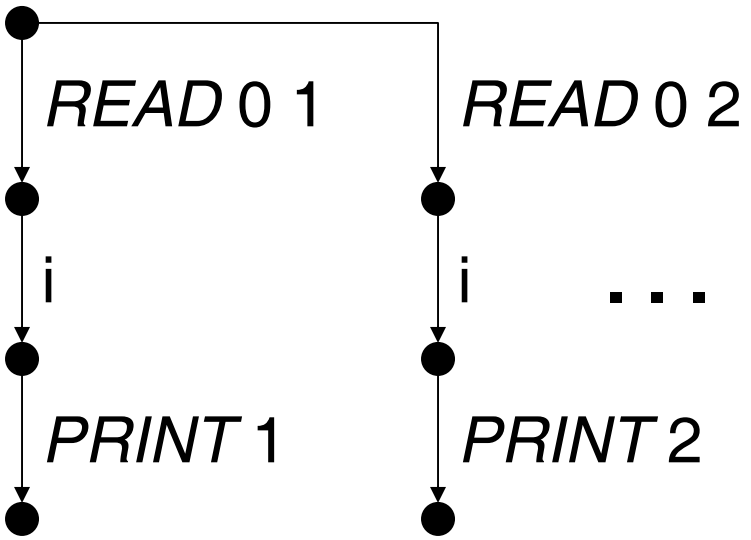
```
(In1; In2; exit
 []
 In2; In1; exit)
>>
(Access; exit)
>>
(Out1; Out2; stop
 []
 Out2; Out1; stop)
```



# Example (2/4)

- Sequential composition with value-passing:

```
READ ?m,n:Nat ;
( [ m >= n ] -> exit (m)
  []
  [ m < n ] -> exit (n) )
>>
accept max:Nat in
PRINT !max ; stop
```



# Example (3/4)

- Definition of terminating process:

```
process Login [LogReq, LogConf, LogAbort] : exit :=
    LogReq;
    ( i ; LogConf ; exit
      []
      i ; LogAbort ; Login [LogReq, LogConf, LogAbort])
endproc
```

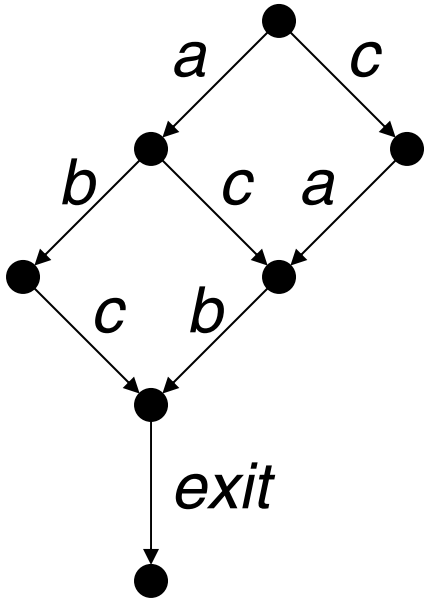
- Example of call:

```
Login [Req,Conf,Abort] >> Transfer ; Logout ; stop
```

# Example (4/4)

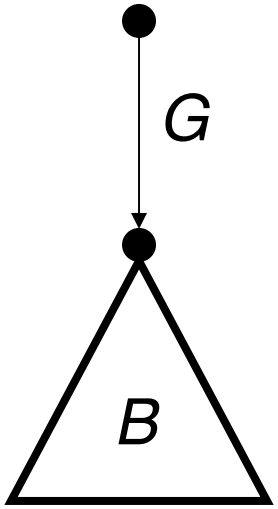
- Combination of "**exit**" and parallel composition: the two behaviours are synchronized on the *exit* gate (they terminate simultaneously)

( *a* ; *b* ; exit ) ||| ( *c* ; exit )

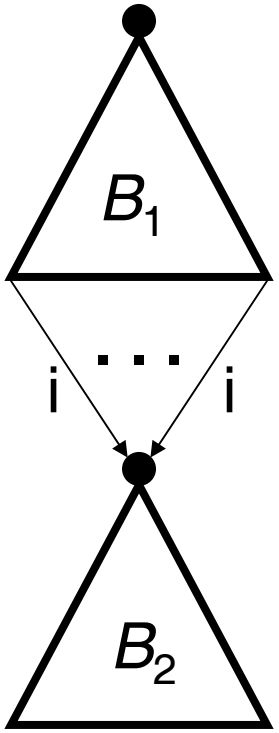


# Sequential composition
(summary)

- In LOTOS, difference between
  “;” (asymmetric)
  and
  “>>” (symmetric):




$G$ ; $B$




$B_1$ >> $B_2$

# Process call

- Let a process *P* defined by:

  **process** *P* [$G_1$, …, $G_n$] ($X_1$:$S_1$, …, $X_n$:$S_n$) :=

  *B*

  **endproc**

- Semantics of a call to *P*:

$$\frac{B\ [\ g_1\ /\ G_1, \ldots, g_n\ /\ G_n\ ]\ [\ v_1\ /\ X_1\ , \ldots, v_n\ /\ X_n\ ] \rightarrow_L B'}{P\ [g_1, \ldots, g_n]\ (v_1, \ldots, v_n) \rightarrow_L B'}$$

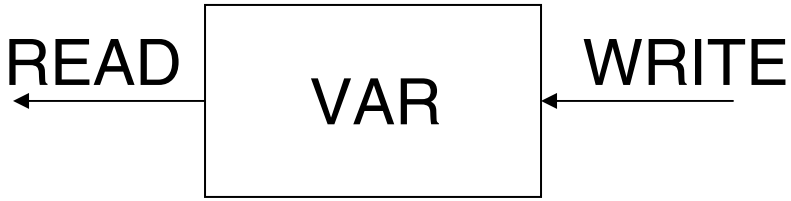
- This semantics explains why a call to

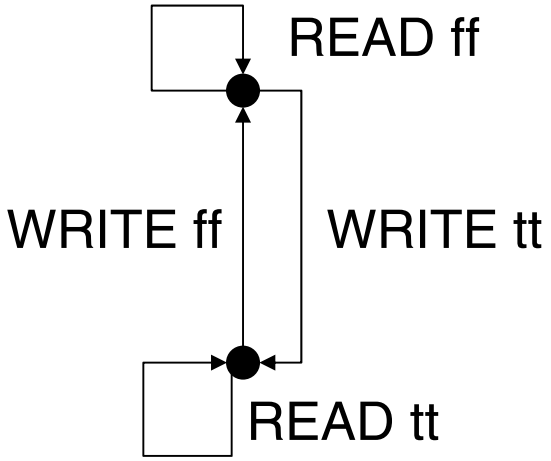
  **process** P[G] : **noexit** := P[G] **endproc**

  is equivalent to **stop**.

# Example

- Boolean variable:




```
process VAR [READ, WRITE] (b:Bool) : noexit :=
  READ !b;
     VAR [READ, WRITE] (b)
  []
  WRITE ?b2:Bool;
     VAR [READ, WRITE] (b2)
endproc
```

# Static semantics
(summary)

- Scope of variables inside behaviours:

| | |
|---|---|
| $B$ ::= $G$ !$V_0$ ?$X$:$S$ … [ $V$ ] ; $B_0$ | $p\ (X) = \{\ V,\ B_0\ \}$ |
| \| hide $G$ in $B_0$ | $p\ (G) = \{\ B_0\ \}$ |
| \| let $X$:$S$ = $V$ in $B_0$ | $p\ (X) = \{\ B_0\ \}$ |
| \| choice $X$:$S$ [] $B_0$ | $p\ (X) = \{\ B_0\ \}$ |
| \| $B_1$ >> accept $X$:$S$ in $B_0$ | $p\ (X) = \{\ B_0\ \}$ |

- Scope of process parameters:

| | |
|---|---|
| process P [G] (X:S) := | $p\ (G) = \{\ B_0\ \}$ |
| $B_0$ | $p\ (X) = \{\ B_0\ \}$ |
| endproc | |

# LOTOS specification

- A LOTOS specification is similar to a process definition:

**specification** Protocol [ SEND, RECEIVE ] : **noexit** :=
  (* ... type definitions *)
 **behaviour**
  (* ... behaviour = body of the specification *)
 **where**
  (* ... process definitions *)
**endspec**

# Example: Peterson's mutual exclusion algorithm

**var** d0 : bool := false { read by P1, written by P0 }
**var** d1 : bool := false { read by P0, written by P1 }
**var** t ∈ {0, 1} := 0 { read/written by P0 and P1}

**loop forever** { P0 }
1 : { ncs0 }
2 : d0 := true
3 : t := 0
4 : **wait** (d1 = false **or** t = 1)
5 : { cs0 }
6 : d0 := false
**endloop**

**loop forever** { P1 }
1 : { ncs1 }
2 : d1 := true
3 : t := 1
4 : **wait** (d0 = false **or** t = 0)
5 : { cs1 }
6 : d1 := false
**endloop**

# Description of variables d0, d1

- Each variable: instance of the same process D
- Current value of the variable: parameter of D
- Reading and writing: RdV on gates R et W

```
process D [R, W] (b:Bool) : noexit :=
    R !b ; D [R, W] (b)
    []
    W ?b2:Bool ; D [R, W] (b2)
endproc
```

- d0 ≡ D [R0, W0] (false), d1 ≡ D [R1, W1] (false)

# Description of variable t

- Variable t: instance of process T
- Current value of the variable: parameter of T
- Reading and writing: RdV on gates R et W

```
process T [R, W] (n:Nat) : noexit :=
    R !n ; T [R, W] (n)
    []
    W ?n2:Bool ; T [R, W] (n2)
endproc
```

- t ≡ T [RT, WT] (0)

# Description of processes P0 and P1

- Process $P_m$: instance of the same process P
- Index m of the process: parameter of P

```
process P [Rm, Wm, Rn, Wn, RT, WT, NCS, CS]
                (m:Nat) : noexit :=
    NCS !m ; Wm !true ; WT !m ;
        P2 [Rm, Wm, Rn, Wn, RT, WT, NCS, CS] (m)
endproc
```

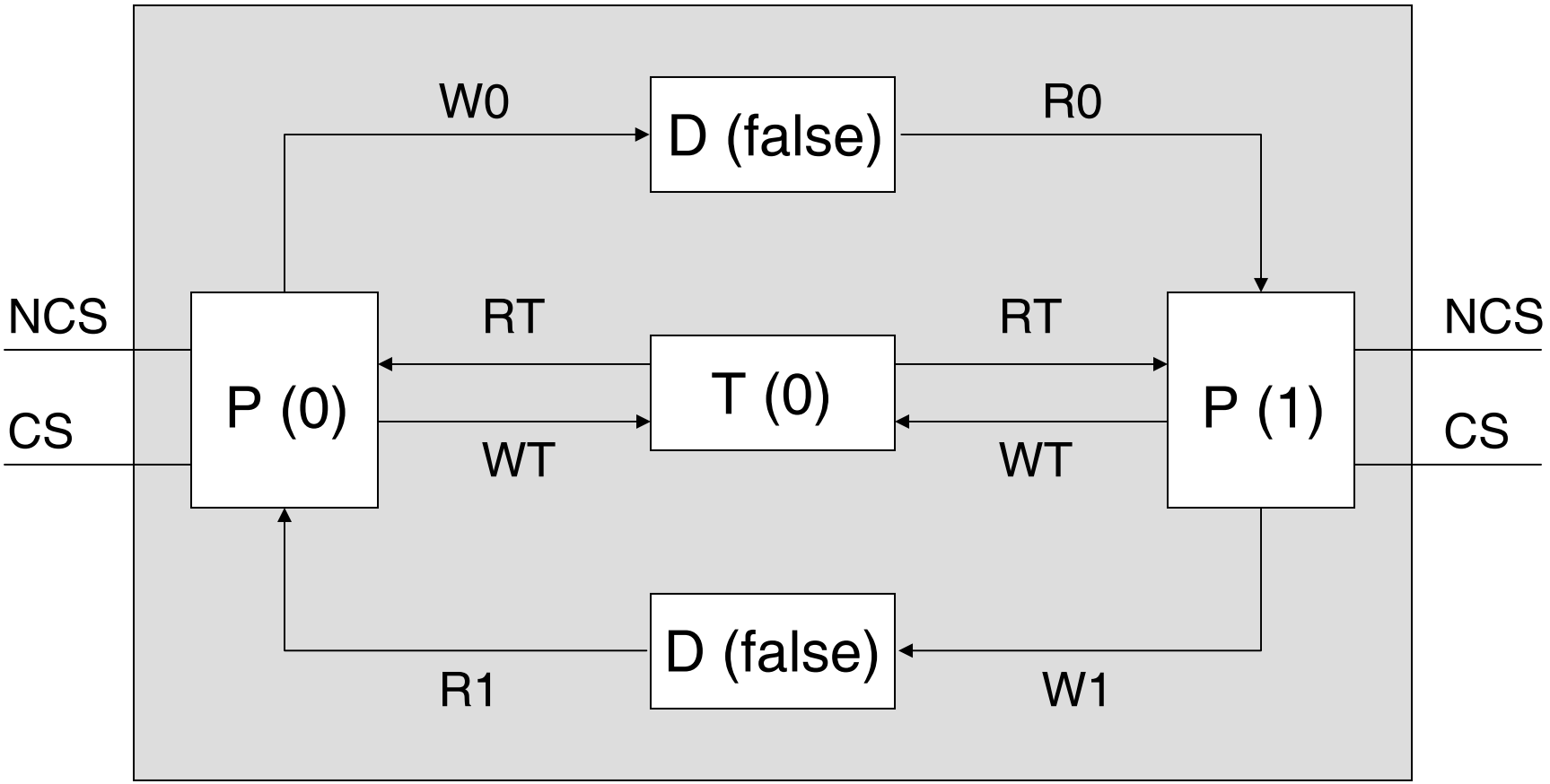
- P0 ≡ P [R0, W0, R1, W1, RT, WT, NCS, CS] (0)
- P1 ≡ P [R1, W1, R0, W0, RT, WT, NCS, CS] (1)

# Processes P0 et P1
(continued)

- Auxiliairy process to describe busy waiting:

```
process P2 [Rm, Wm, Rn, Wn, RT, WT, NCS, CS]
                (m:Nat) : noexit :=
   Rn ?dn:Bool ; RT ?t:Nat ;
   ( [ dn and (t eq m) ] ->
        P2 [Rm, Wm, Rn, Wn, RT, WT, NCS, CS] (m)
     []
     [ not (dn) or (t eq ((m + 1) mod 2)) ] ->
        CS !m ; Wn !false ;
        P [Rm, Wm, Rn, Wn, RT, WT, NCS, CS] (m) )
endproc
```

# Architecture of the system
(graphical)

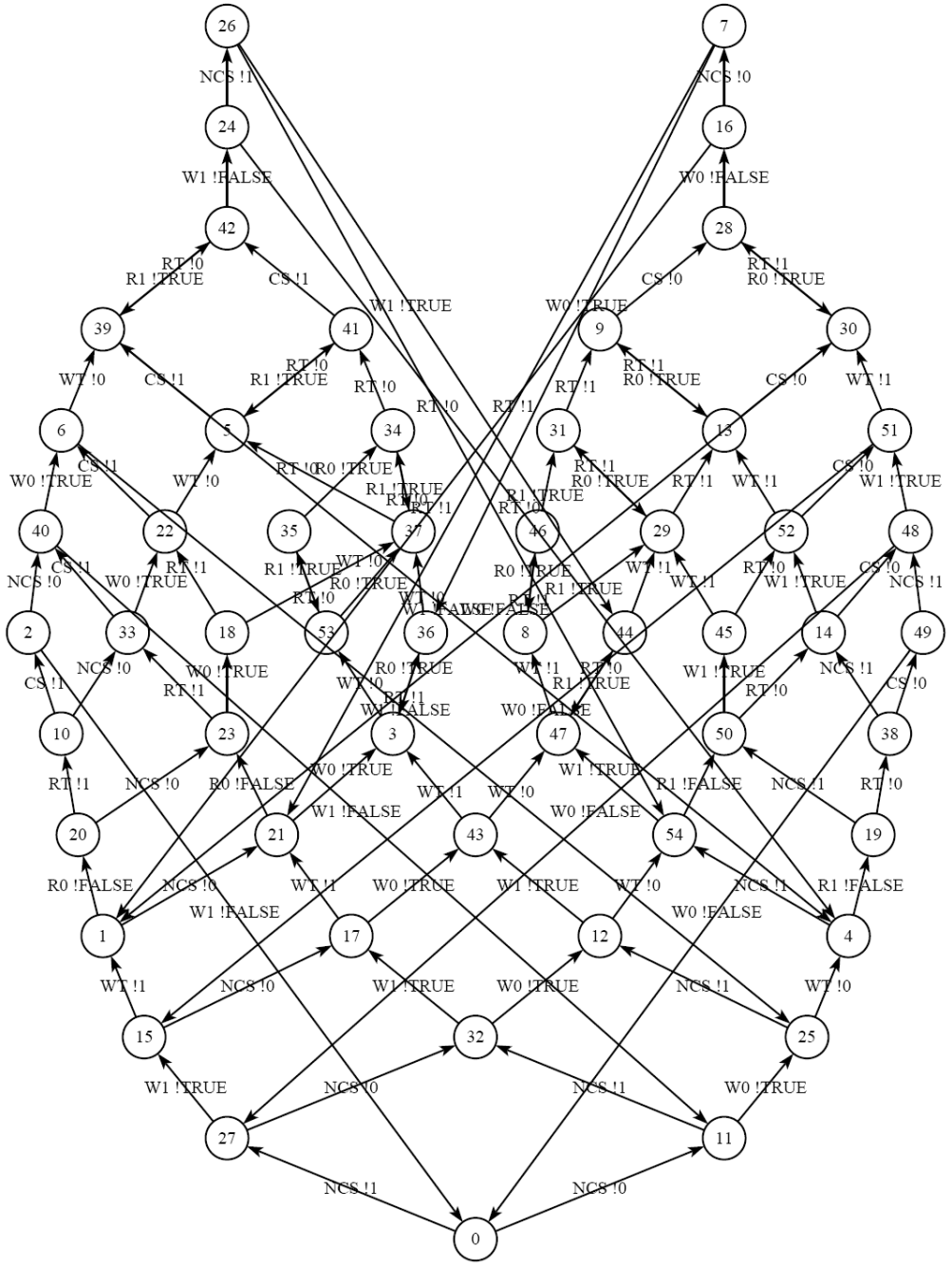


# Architecture of the system
(textual)

```
hide R0, W0, R1, W1, RT, WT in
    (
        P [R0, W0, R1, W1, RT, WT, NCS, CS] (0)
        |||
        P [R1, W1, R0, W0, RT, WT, NCS, CS] (1)
    )
    |[ R0, W0, R1, W1, RT, WT ]|
    (
        T [RT, WT] (0)
        |||
        D [R0, W0] (false)
        |||
        D [R1, W1] (false)
    )
```

# LTS model

- 55 states
- 110 transitions



# Process algebraic languages
(summary)

- More concise than communicating automata: process parameterization, value-passing communication (Exercise: model variables d0, d1, t using a single gate allowing both reading / writing)
- In general, there are several ways of describing the parallel composition of processes (Exercise: write a different expression for the architecture of Peterson's algorithm)
- Modeling of nested loops: mutually recursive LOTOS processes (Exercise: model processes P0, P1 using a single LOTOS process)
- But: E-LOTOS process part is much more convenient