Part I: Rewriting Models of Boolean Programs

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Software model-checking

Big research challenge of the 00s: extension of model checking techniques to ‘high-level’ software.

Three main research questions:

- Integration of the tools in the software development process.
  - Users trust their hardware but may not trust their software: “post-mortem” verification, “backstage” verification tools . . .
- Automatic extraction of models from code.
- Algorithms for infinite-state systems.
  - Software systems are very often infinite-state.

A “lazy” approach to software verification

Construct a sequence of increasingly faithful models that under- or overapproximate the code.

Underapproximations: 32-bit integer $\longrightarrow$ 2-bit integer, 500MB heap $\longrightarrow$ 10B heap.

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> Associate to each predicate a boolean variable.
> Example: $x < y \mapsto a$ $\quad$ $x = 0 \mapsto b$

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Overapproximate by a program over these variables.

Example: $x := y$ is overapproximated by

$a := \mathrm{false};$

$\textbf{if } (a \textbf{ and } b) \textbf{ then } b := \mathrm{false}$

$\textbf{else } b := \mathrm{true} \textbf{ or } \mathrm{false}$

Both under- and overapproximations are boolean programs:

Same control-flow structure as code + possibly nondeterminism.

Only one datatype: booleans.

Conceptually could also take any enumerated type but booleans are the bridge to SAT and BDD technology.

Rewriting models of boolean programs

Boolean programs are still pretty complicated objects:

- Procedures/methods and recursion.
- Concurrency and communication (threads, cobegin-coend sections).
- Object-orientation.

Must be “compiled” into simpler and formal models.

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Use rewriting to model boolean programs. In a nutshell:

- Model program states as terms.
- Model program instructions as term-rewriting rules.
- Model program executions as sequences of rewriting steps.

Fundamental analysis problem: Reachability

But reachability between two states not enough for verification purposes

Safety properties often characterized by an infinite set of dangerous states.

Set of initial states also possibly infinite.

Generalized reachability problem: Given two (possibly infinite) sets I and D of initial and dangerous states, respectively, decide if some state of D is reachable from some state of I.

Challenge: Find a finite ("symbolic") representation of the (possibly infinite) set of states reachable or backward reachable from a given (possibly infinite) set of states.

- $pre^*(S)$ denotes the set of predecessors of S.
  (states backward reachable from states in S)
- $post^*(S)$ denotes the set of successors of S.
  (states forward reachable from states in S)

Strategies: Compute $pre^*(D)$ and check if $I \cap pre^*(D) = \emptyset$, or compute $post^*(I)$ and check if $post^*(I) \cap D = \emptyset$

Program for the rest of Part I

Rewriting models for:

- Procedural sequential programs.
- Multithreaded while-programs.
- Multithreaded procedural programs.
- Procedural programs with cobegin-coend sections.

For each of those:

- Complexity of the reachability problem.
- Finite representations for symbolic reachability.

A rewriting model of procedural sequential programs

State of a procedural boolean program: $(\, g, \ell, n, (\ell_1, n_1) \ldots (\ell_k, n_k) \,)$, where

- g is a valuation of the global variables,
- ℓ is a valuation of local variables of the currently active procedure,
- n is the current value of the program pointer,
- l_i is a saved valuation of the local variables of the caller procedures, and
- n_i is a return address.

Modelled as a string $g \, \langle \ell, n \rangle \, \langle \ell_1, n_1 \rangle \, \ldots \, \langle \ell_k, n_k \rangle$

Instructions modelled as string-rewriting rules, e.g. $t \, \langle t, m_0 \rangle \longrightarrow f \langle f\, t\, f, p_0 \rangle \, \langle t, m_1 \rangle$

Prefix-rewriting policy:

$$\frac{u \longrightarrow w}{u\, v \xrightarrow{r} w\, v}$$

An example

bool function *foo*(ℓ)
f_0: **if** ℓ **then**
f_1: **return** false
else
f_2: **return** true
fi

procedure *main*()
global b
m_0: **while** b **do**
m_1: $b := foo(b)$
od;
m_2: **return**

$$
\begin{array}{rcl}
b\,\langle t, f_0\rangle & \longrightarrow & b\,\langle t, f_1\rangle \\
b\,\langle f, f_0\rangle & \longrightarrow & b\,\langle f, f_2\rangle \\
b\,\langle \ell, f_1\rangle & \longrightarrow & f \\
b\,\langle \ell, f_2\rangle & \longrightarrow & t
\end{array}
$$

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\begin{array}{rcl}
t\, m_0 & \longrightarrow & t\, m_1 \\
f\, m_0 & \longrightarrow & f\, m_2 \\
b\, m_1 & \longrightarrow & b\langle b, f_0\rangle\, m_0 \\
b\, m_2 & \longrightarrow & \epsilon
\end{array}
$$

(b and ℓ stand for both t and f)

Prefix string rewriting. From theory . . .

First studied by Büchi in 64 under the name regular canonical systems as a variant of semi-Thue systems.

Theorem: Given an effectively regular (possibly infinite) set S of strings, the sets $pre^*(S)$ and $post^*(S)$ are also effectively regular.

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Polynomial algorithms by Bouajjani, E., Maler and Finkel, Willems, Wolper in 97.

- Saturation algorithms: the automata for $pre^*(S)$ and $post^*(S)$ are essentially obtained by adding transitions to the automaton for S.
  (Algorithms for similar models by Alur, Etessami, Yannakakis, and Benedikt, Godefroid, Reps and . . .)

. . . to applications

Efficient algorithms by E., Hansel, Rossmanith and Schwoon in 00.

Theorem (informal): Let Σ, R be the alphabet and set of rules of a 2-normalized prefix-rewriting system system and let A be a “small” NFA over Σ.
An NFA for $post^*(L(A))$ can be constructed in $O(|\Sigma||R|^2)$ time and space.
An NFA for $pre^*(L(A))$ can be constructed in $O(|\Sigma|^2|R|)$ time and $O(|\Sigma||R|)$ space.

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jMOPED by Suwimonteerabuth and Schwoon in 2005

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Here:

Regular canonical systems, introduced by Büchi in the early 60s because he liked them, have been translated, unlikely as it may seem, into effective algorithms for software model checking tools.

A rewriting model of multithreaded while-programs

Communication through global variables.

State determined by: $\{\ g, (\ell_0, n_0), (\ell_1, n_1) \ldots (\ell_k, n_k)\ \}$ where

- g is a valuation of the global variables,
- ℓ_i is a valuation of the local variables of the i-th thread, and
- n_i is the value of the program pointer of the i-th thread.

Modelled as a multiset

$$g \parallel \langle \ell_0, n_0 \rangle \parallel \langle \ell_1, n_1 \rangle \parallel \ldots \parallel \langle \ell_k, n_k \rangle$$

Instructions modelled as multiset-rewriting rules, e.g.

$$t\, f \parallel m_0 \to f\, f \parallel m_1 \parallel \langle f, p_0 \rangle$$

Multiset rewriting, or rewriting modulo assoc. and comm. of $\parallel$.

An example

thread $p()$

p_0:	**if** ? **then**	$b \parallel p_0$	$\longrightarrow$	$b \parallel p_1$
p_1:	$b := \text{true}$	$b \parallel p_0$	$\longrightarrow$	$b \parallel p_2$
	else	$b \parallel p_1$	$\longrightarrow$	$t \parallel p_3$
p_2:	$b := \text{false}$	$b \parallel p_2$	$\longrightarrow$	$f \parallel p_3$
	fi;			
p_3:	**end**	$b \parallel p_3$	$\longrightarrow$	ϵ

thread $main()$
global b

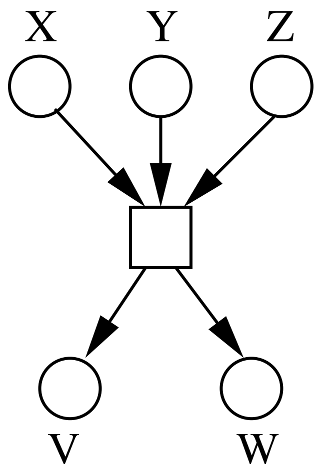
m_0:	**while** b **do**	$t \parallel m_0$	$\longrightarrow$	$t \parallel m_1$
m_1:	**fork** $p()$	$f \parallel m_0$	$\longrightarrow$	$f \parallel m_2$
	od;	$b \parallel m_1$	$\longrightarrow$	$b \parallel m_0 \parallel p_0$
m_2:	**end**	$b \parallel m_2$	$\longrightarrow$	ϵ

Multiset rewriting

Theorem [Mayr, Kosaraju, Lipton, 80s]: The reachability problem for multiset-rewriting is decidable but EXPSPACE-hard.

- Equivalent to the reachability problem for Petri nets.
- A place for each alphabet letter.
- A Petri net transition for each rewrite rule.

$$X \parallel Y \parallel Z \longrightarrow V \parallel W$$



Algorithms (not only proofs) quite complicated.

Negative results for $pre^*(\{s\})$ and $post^*(\{s\})$.

Symbolic reachability for pre^* and upward-closed sets

Upward-closed set: if some multiset t belongs to the set, then $t \parallel t'$ also belongs to the set for every t'.

Finitely representable e.g. by the its of minimal elements.

Upward-closed sets capture properties that can be decided by inspecting a bounded number of threads (e.g. mutual exclusion).

Theorem [Abdulla et al. 96]: Given a multiset-rewriting system and an upward-closed set of states S, the set $pre^*(S)$ is upward-closed and effectively constructible.

- Very simple algorithm: compute $pre(S), pre^2(S), pre^3(S) \ldots$.

Extensions applied to multithreaded Java [Delzanno, Raskin, Van Begin 04].

Monadic multiset-rewriting

Monadic rules $\equiv$ no global variables $\equiv$ no communication

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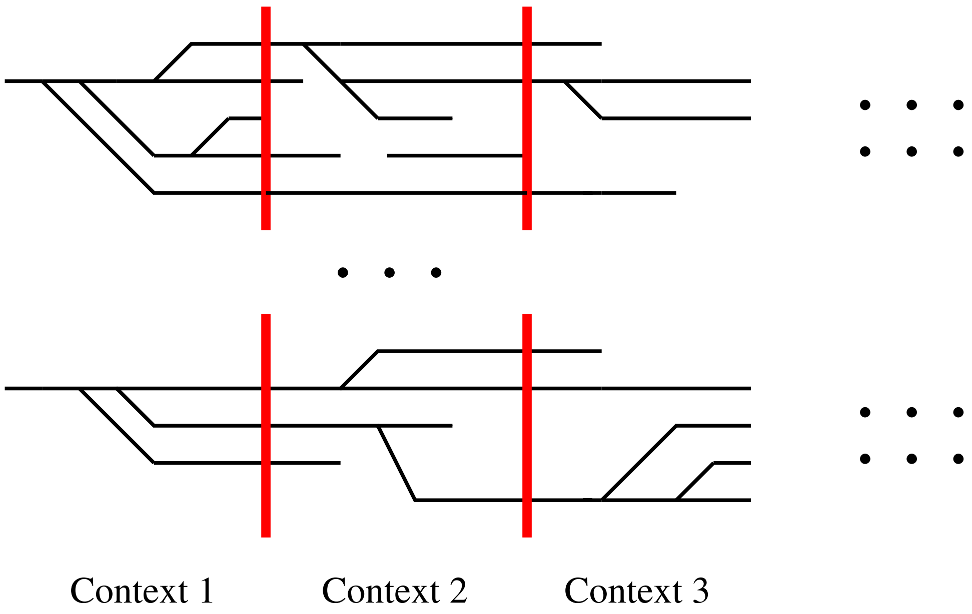
They are good for underapproximations [Qadeer and Rehof 05]



Reachability

Theorem [Huyhn 85, E.95]: The reachability problem for monadic multiset-rewrite systems is NP-complete.

- Membership in NP not completely trivial.
- Hardness very easy, reduction from SAT:

  A thread for each variable x_i that (a) nondeterministically chooses $l_i \in \{x_i, \overline{x}_i\}$ and (b) spawns a clause thread for each clause satisfied by l_i.

  The thread for a clause does nothing and terminates.

  Formula satisfiable iff there is state at which one thread per clause is active.

Symbolic reachability for semi-linear sets

Semi-linear sets usually defined as subsets of $\mathbb{N}^n$.

- Finite union of linear sets.
- $\{r + \lambda_1 p_1 + \ldots + \lambda_n p_n \mid \lambda_1, \ldots, \lambda_n \in \mathbb{N}\}$.

Language interpretation: “commutative closure” of the regular languages.

Similar properties to regular languages: closure under boolean operations, decidable (but no longer polynomial) membership problem, etc.

Theorem [E.95]: Given a monadic multiset-rewriting system and a semi-linear set of states S, the sets $post^*(S)$ and $pre^*(S)$ are semi-linear and effectively constructible.

Multithreaded procedural programs

Two-counter machines can be simulated by a program with two recursive threads communicating over two global (boolean) variables:

- Tops of the recursion stacks contains two copies of the machine's control point.
- Depths the two recursion stacks model the values of the counters.
- Calls and returns model increasing and decrementing the counters.
- One variable to ensure alternation of moves.
- One variable to keep the two copies of the control point "synchronized".

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Communication-free case: [Bouajjani, Müller-Olm and Touili 05]

Communication through nested locks: [Kahlon and Gupta 06, Kahlon 09]

A rewriting model for the communication-free case

State of a multithreaded procedural program without global variables: multiset $\{s_1, s_2 \ldots, s_k\}$ of states of procedural programs, where

$$s_i = (\ell_{i0}, n_{i0})\,(\ell_{i1}, n_{i1}) \ldots (\ell_{im}, n_{im})$$

Modelled as a string $\# w_k \# w_{k-1} \# \ldots \# w_1$ where

$$w_i = \langle \ell_{i0}, n_{i0} \rangle \, \langle \ell_{i1}, n_{i1} \rangle \, \ldots \, \langle \ell_{im}, n_{im} \rangle$$

Instructions modelled as string-rewriting rules. A new thread is inserted to the left of its creator, e.g.

$$\# \, \langle b, m_1 \rangle \longrightarrow \# \, p_0 \, \# \, \langle f, m_3 \rangle$$

Threads "in the middle" of the string should also be able to "move": back to ordinary rewriting

$$\frac{u \longrightarrow w}{v_1 \, u \, v_2 \xrightarrow{\;r\;} v_1 \, w \, v_2}$$

An example

process $p()$;
p_0: **if** (?) **then**
p_1: **call** $p()$
else
p_2: **skip**
fi;
p_3: **return**

$$\# p_0 \longrightarrow \# p_1$$
$$\# p_0 \longrightarrow \# p_2$$
$$\# p_1 \longrightarrow \# p_0\, p_3$$
$$\# p_2 \longrightarrow \# p_3$$
$$\# p_3 \longrightarrow \#$$

process $main()$
m_0: **if** (?) **then**
m_1: **fork** $p()$
else
m_2: **call** $main()$
fi;
m_3: **return**

$$\# m_0 \longrightarrow \# m_1$$
$$\# m_0 \longrightarrow \# m_2$$
$$\# m_1 \longrightarrow \# p_0 \# m_3$$
$$\# m_2 \longrightarrow \# m_0\, m_3$$
$$\# m_3 \longrightarrow \# \epsilon$$
$$\#\# \longrightarrow \#$$

Analysis

Theorem [BMOT05]: For every effectively regular set S of states, the set $pre^*(S)$ is regular and a finite-state automaton recognizing it can be effectively constructed in polynomial time.

- Similar to pre^* for monadic string-rewriting [Book and Otto 93].

Theorem [BMOT05]: For every effectively context-free set S of states, the set $post^*(S)$ is context-free and a pushdown automaton recognizing it can be effectively constructed in polynomial time.

Counterexample to regularity: P that spawns a copy of Q and calls itself.

The number of threads is equal to the depth of the recursion.

Reachability set: $\{(\#q)^n \# p^{(n+1)} \mid n \geq 0\}$.

Cobegin-coend sections

Difference with threads: implicit synchronization induced by the coend.

- “Threads have to wait for its siblings to terminate.”
- Corresponds to calling procedures in parallel.

Rewriting model only works well for the communication-free (monadic) case.

States modelled as terms with both $\|$ and $\cdot$ as infix operators e.g

$$(\langle t, p_1 \rangle \parallel q_2) \cdot \langle t\, f, m_1 \rangle$$

Rewriting modulo assoc. of $\cdot$ and assoc. and comm. of $\|$.

This model is called monadic process rewrite systems (monadic PRS) [Mayr 00].

Analysis

Symbolic reachability with commutative hedge automata (CHA) [Lugiez 03].

Theorem [Bouajjani and Touili 05]: Given a monadic PRS, for every CHA-definable set of terms T, the sets $post^*(T)$ and $pre^*(T)$ are CHA-definable and effectively constructible.

Weaker approach: construct not the sets $post^*(T)$ or $pre^*(T)$ themselves, but representatives w.r.t. the equational theory.

Sufficient for control reachability problems.

Theorem [Lugiez and Schnoebelen 98, E. and Podelski 00]:
Let R be a monadic PRS and let A be a bottom-up tree automaton.
One can construct in $O(|R| \cdot |A|)$ time bottom-up tree automata recognizing a set of representatives of $post^*(L(A))$ and $pre^*(L(A))$.

Conclusions

Rewriting concepts can be used to give elegant semantics to programming languages.

- String/multiset rewriting correspond to sequential/parallel computation.
- Monadic/non-monadic rewriting correspond to absence or presence of communication.
- Rewriting modulo useful for combining concurrency and procedures.

Symbolic reachability is the key problem to solve.

Comparison with process algebras:

- Process algebras have a notion of hiding or encapsulation.
- Rewriting much closer to automata theory $\longrightarrow$ algorithms.