Lecture 1:
Verification of Concurrent Programs
Part 1: Decidability and Complexity Results

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VTSA, MPI-Saarbrücken, September 2012

Outline of the lectures

- Lecture 1: Concurrent programs: Decidability and complexity Results
  - Basic models
  - Limits of the decidability of the reachability problem
  - Classes of programs/models with a decidable state reachability problem
- Lecture 2: Concurrent programs: Under-approximate analysis
  - Bounded analysis for concurrent programs
  - Decidability and complexity issues
  - Compositional reduction to state reachability in *sequential programs*
- Lecture 3: Weak memory models: State reachability problem
  - Weaker models than Sequential Consistency
  - (Un)Decidability and complexity of the state reachability problem
  - Efficient under-approximate analysis: Reduction to SC state reachability
- Lecture 4: Weak memory models: Robustness against a WMM
  - Check that all behaviors are still sequentially consistent
  - Decidability and complexity
  - Reduction to SC state reachability

Concurrent Programs

- Parallel threads (with/without procedure calls)
- Static/Dynamic number of threads
- Communication
  - Shared memory
    - Notion of action atomicity
    - Actions by a same threads are executed in the same order (Sequential Consistency)
    - Actions by different threads are interleaved non-deterministically
  - Message passing
    - Channels (queues)
    - Unordered/FIFO ...
    - Perfect/Lossy
- We assume finite data domain (e.g., booleans).

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- Product grows exponentially in # threads and # variables.
- Reachability is decidable, and PSPACE-complete.
  [Kozen, FOCS'77]

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Facing the state-space explosion

- Partial order techniques
  - Independent actions ⇒ commutable actions ⇒ many interleavings
  - Explore representatives up to independent actions commutations
    Godefroid, Wolper, Peled, Holzman, Valmari, ...
- Symbolic techniques
  - Compact representations of sets of states + fixpoint calculations
  - Bounded model checking + SAT solvers
    Clarke, McMillan, Somenzi, Biere, Cimatti, ...

Beyond the finite-state case

- Unbounded (parametric/dynamic) number of threads
  - Undecidable in general if threads Ids are allowed
  - $\Rightarrow$ Anonymous threads
- Unbounded channels
  - Undecidable in general in case of FIFO queues
  - $\Rightarrow$ Unordered queues (multisets), lossy queues

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- Safety is reducible to state reachability in VASS / Coverability in PN

Vector Addtion Systems with States

- Finite state machine + finite number of counter $C = \{c_1, \ldots, c_n\}$.
- Operations: (No test to zero)
  - $c_i := c_i + 1$
  - $c_i > 0 \ / \ c_i := c_i - 1$
- Configuration: (q, V) where q is a control state and $V \in \mathbb{N}^n$
- Initial configuration: $(q_0, \mathbf{0})$ where $\mathbf{0} = 0^n$.
- Transition relation:

$$(q_1, V_1) \xrightarrow{op} (q_2, V_2) \ \textit{iff}$$

  - $op =$ "$c_i := c_i + 1$", *and* $V_2 = V_1[c_i \leftarrow (V_1(c_i) + 1)]$
  - $op =$ "$c_i > 0 \ / \ c_i := c_i - 1$, *and* $(V_1(c_i) > 0$ *and* $V_2 = V_1[c_i \leftarrow (V_1(c_i) - 1)])$

From Multithreaded Programs to VASS

- Associate a control state with each valuation of the globals
- Associate a counter with each valuation of thread locals
- A statement moving globals from g to g' and locals from ℓ to ℓ':

$$g \xrightarrow{c_\ell > 0 / c_\ell := c_\ell - 1 \,;\, c_{\ell'} := c_{\ell'} + 1} g'$$

- Creation of a new thread at initial state ℓ:

$$g \xrightarrow{c_\ell := c_\ell + 1} g$$

VASS: Reachability Problems

- State reachability problem:

  Given a state q, determine if a configuration (q, V) is reachable, for some $V \in \mathbb{N}^n$ (any one).

- Coverability problem:

  Given a configuration (q, V), determine if a configuration (q, V') is reachable, for some $V' \geq V$. (We say that (q, V) is coverable*.)*

EXSPACE-complete [Rackoff 78]

NB: *Coverability can be reduced to State reachability and vice-versa.*

- Configuration reachability problem:

  Determine if a given configuration (q, V) is reachable.

Decidable [Mayr 81], [Kosaraju 82].
EXPSPACE-hard [Lipton 75]. No upper bound known.

Well Structured Systems

[Abdulla et al. 96], [Finkel, Schnoebelen, 00]

- Let U be a universe.
- Well-quasi ordering $\preceq$ over U: $\forall c_0, c_1, c_2, \ldots, \ \exists i < j, \ c_i \preceq c_j$
- $\Rightarrow$ Each (infinite) set has a finite minor set.

- Let $S \subseteq U$. Upward-closure $\overline{S}$ = minimal subset of U s.t.
  - $S \subseteq \overline{S}$,
  - $\forall x, y. \ (x \in S \text{ and } x \preceq y) \Rightarrow y \in \overline{S}$.
- A set is upward closed if $\overline{S} = S$
- Upward closed sets are definable by their minor sets
  - Assume there is a function *Min* which associates a minor to each set.
  - Assume $pre(Min(S))$ is computable for each set S.

- Monotonicity: $\preceq$ is a simulation relation

$$\forall c_1, c_1', c_2. \ \left((c_1 \longrightarrow c_1' \text{ and } c_1 \preceq c_2) \Rightarrow \exists c_2'. \ c_2 \longrightarrow c_2' \text{ and } c_1' \preceq c_2'\right)$$

Key lemma

Lemma

The pre and pre images of upward closed set are upward closed*

1. Let S be an upward closed set.
2. Assume $pre(S)$ is not upward closed.
3. Let $c_1 \in pre(S)$, and let $c_2 \in U$ such that $c_1 \preceq c_2$ and $c_2 \notin pre(S)$

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8. For pre^*: the union of upward closed sets is upward closed.

Backward Reachability Analysis

Consider the increasing sequence $X_0 \subseteq X_1 \subseteq X_2 \ldots$ defined by:

- $X_0 = Min(S)$
- $X_{i+1} = X_i \cup Min(pre(\overline{X_i}))$

Termination:

There is a index $i \geq 0$ *such that* $X_{i+1} = X_i$

- The set $pre^*(S)$ is upward closed $\Rightarrow$ has a finite minor
- Wait until a minor is collected
- How long shall we wait?
- Non primitive recursive in general

The case of VASS

- Usual $\leq$ order over $\mathbb{N}$ is a WQO (Dickson lemma)
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- Upward-closed sets = finite disjunctions of $\bigwedge_{i=1}^{n} l_i \leq c_i$, where $l_i \in \mathbb{N}$
- Computation of the Pre:
  - $op =$ "$c_j := c_j + 1$" : $(\bigwedge_{i \neq j} l_i \leq c_i) \wedge (max(l_j - 1, 0) \leq c_j)$
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- Coverability is still decidable. (But not reachability. [Dufourd et al. 98])

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- Is configuration reachability decidable ?
- Yes, lossyness $\Rightarrow$ (reachability $\simeq$ coverability)

Concurrent Programs with Procedures

- Procedural program $\rightarrow$ Pushdown System (finite control + stack)
- Concurrent program $\rightarrow$ Concurrent PDS's (Multistack systems)
- Two stacks can simulate a Turing tape.
- Concurrent programs with 2 threads are Turing powerful.
- $\Rightarrow$ Restrictions
  - Classes of programs with particular features
  - Particular kind of behaviors
    (under-approximate analysis for bug detection)

Asynchronous Programs

- Synchronous calls

  Usual procedure calls

- Asynchronous calls
  - *Calls are stored and dispatched later by the scheduler*
  - *They can be executed in any order*

- Event-driven programming (requests, responses)
- Useful model: distributed systems, web servers, embedded systems

Formal Models: Multiset Pushdown Systems

- A task is a sequential (pushdown) process with dynamic task creation
- Created tasks are stored in an unordered buffer (multiset)
- Tasks run until completion
- If the stack is empty, a task in moved from the multiset to the stack

Difficulties

- Unbounded buffer of tasks
- The buffer is a multiset $\Rightarrow$ can be encoded as counters
- Need to combine somehow PDS with VASS
- Stack $\Rightarrow$ not Well Structured
- How to get rid of the stack ?

State Reachability of Multiset PDS

Theorem

The control state reachability problem for MPDS is EXPSPACE-complete.

Reduction to/from the coverability problem for Petri.

First decidability proof by K. Sen and M. Viswanathan, 2006

Semi-linear Sets

- Linear set over $\mathbb{N}^n$ is a set of the form

$$\{\vec{u} + k_1\vec{v_1} + \cdots + k_m\vec{v_m} \ : \ k_1, \ldots, k_m \in \mathbb{N}\}$$

  where $\vec{u}, \vec{v_1}, \ldots, \vec{v_m} \in \mathbb{N}^n$

- Semi-linear set = finite union of linear sets.

- Examples:
  - $\{(0,0) + k(1,1) \ : \ k \geq 0\} \ \equiv \ x_1 = x_2$
  - $\{(0,0) + k(1,2) \ : \ k \geq 0\} \ \equiv \ 2x_1 = x_2$
  - $\{(0,3) + k(1,1) \ : \ k \geq 0\} \ \equiv \ x_1 + 3 = x_2$
  - $\{(0,3) + k_1(0,1) + k_2(1,1) \ : \ k \geq 0\} \ \equiv \ x_1 + 3 \leq x_2$
  - $\{(0,0,0) + k_1(1,0,1) + k_2(0,1,1) \ : \ k_1, k_2 \geq 0\} \ \equiv \ x_1 + x_2 = x_3$
  - $\{(0,0,3) + k_1(1,0,2) + k_2(0,1,1) \ : \ k_1, k_2 \geq 0\} \ \equiv \ 2x_1 + x_2 + 3 = x_3$

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- Theorem [Ginsburg, Spanier, 1966]

  A set is semi-linear iff it is definable in Presburger arithmetics.

Parikh's image

- Let $\Sigma = \{a_1, \ldots, a_n\}$.
- Given a word $w \in \Sigma^*$, the *Parikh image* of w is:

$$\phi(w) = (\#_{a_1}(w), \ldots, \#_{a_n}(w)) \in \mathbb{N}^n$$

- Given a language $L \subseteq \Sigma^*$, $\phi(L) = \{\phi(w) \ : \ w \in L\}$
- Examples:
  - $L_1 = \{a^n b^n \ : \ n \geq 0\}$, $\phi(L_1) = \{(x_1, x_2) \ : \ x_1 = x_2\}$
  - $L_2 = \{a^n b^n c^n \ : \ n \geq 0\}$, $\phi(L_2) = \{(x_1, x_2, x_3) \ : \ x_1 = x_2 \wedge x_2 = x_3\}$
  - $L_3 = (ab)^* = \{(ab)^n \ : \ n \geq 0\}$, $\phi(L_3) = \{(x_1, x_2) \ : \ x_1 = x_2\}$

Semi-linear sets, CFL's, and RL's

- Parikh's Theorem (1966)

  For every Context-Free Language L, $\phi(L)$ *is a semi-linear set.*

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  For every Context-Free Language L, $\phi(L)$ is a semi-linear set.

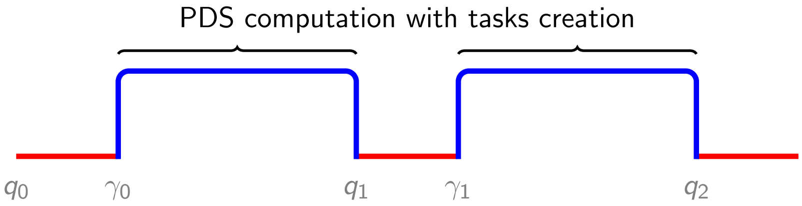
- Proposition

  For every semi-linear set S, there exists a Regular Language L such that $\phi(L) = S$.

- Corollary

  For every Context-Free Language L, there exists a Regular language L' such that $\phi(L) = \phi(L')$.

From Multiset PDS to VASS



Pending tasks Multiset

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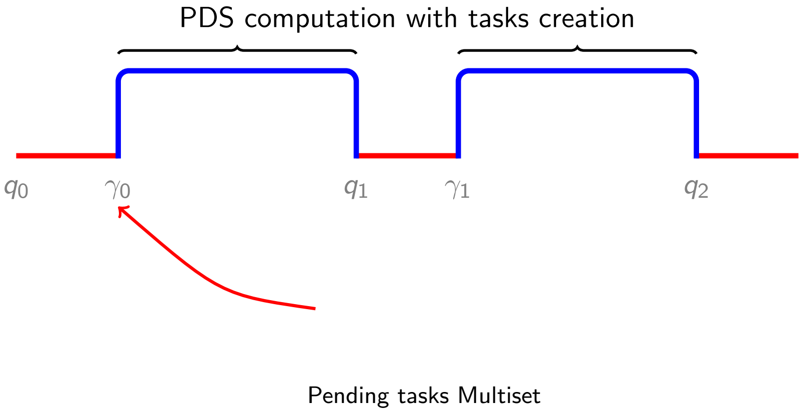

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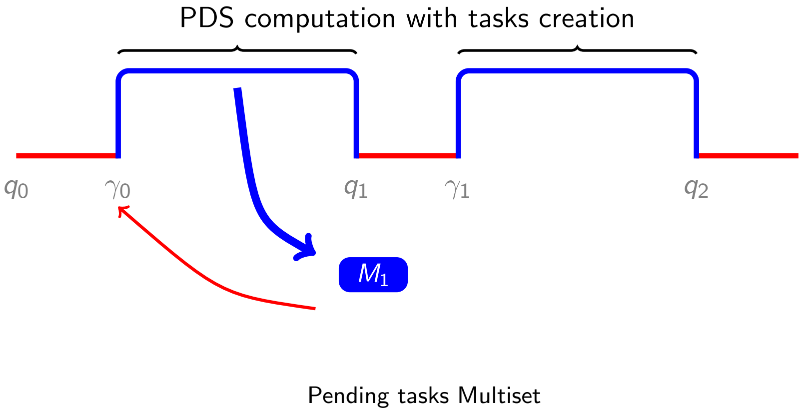

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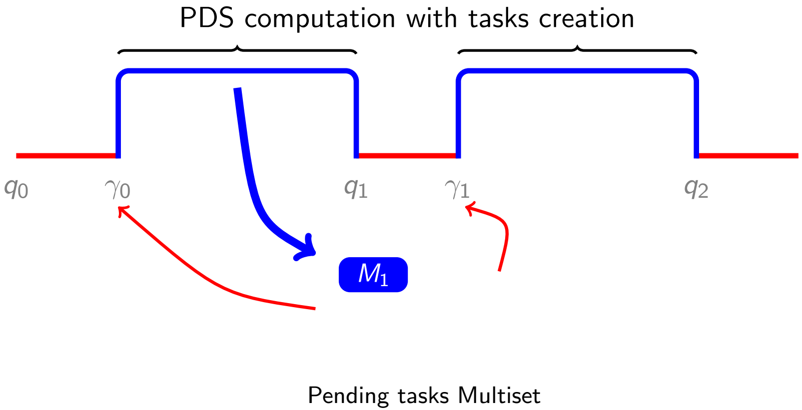

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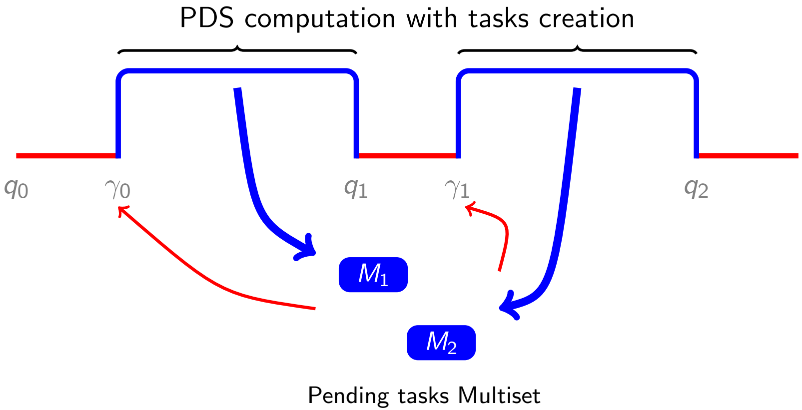

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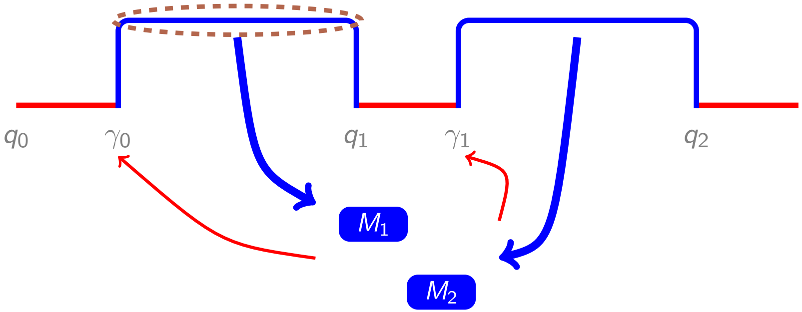

$$q_0, \gamma_0 \overset{L_1}{\Longrightarrow}^* q_1, \epsilon$$

- L_1= Set of sequences of created tasks
- L_1 is a Context-Free Language
- M_1 is the Parikh image of L_1

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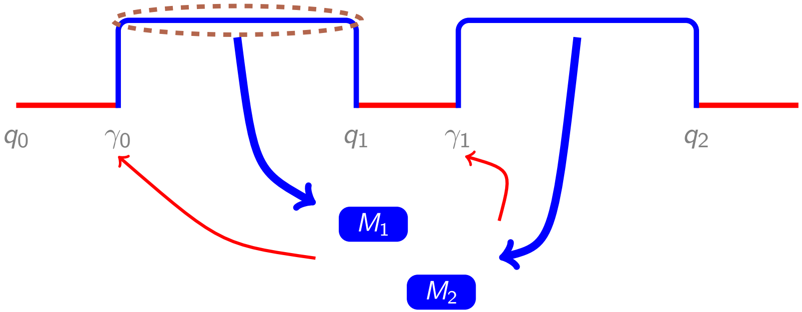

Parikh's Theorem: M_i is definable by a finite state automaton S_i

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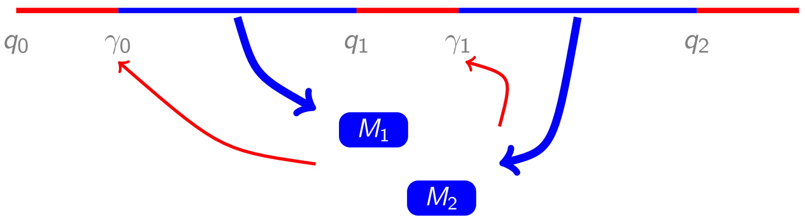

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Construction of a VASS: Simulation of S_i + task consumption rules

Message-Passing Programs with Procedures

- Undecidable even for bounded channels
- Restrictions on
  - Interaction between recursion and communication (e.g., communication with empty stack)
  - Kind of channels (e.g., lossy, unordered)
  - Topology of the network
- Decidable classes

  [La Torre et al. TACAS'08], [Atig et al., CONCUR'08], ...

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- At the end, we need to solve reachability in one pushdown system $\widetilde{P_n}$

End of Lecture 1:

- Dynamic networks of processes can be represented using VASS
- Procedures make things more difficult
- Constraints on interaction between concurrency and recursion are necessary to get decidable classes
- Asynchronous is an important class of programs for which verification problems are decidable
- Reasoning about interfaces/summaries is an important tool for the design of decision procedures
- Still, complexity is high. Need of efficient techniques.