Dynamic Logic for Practical Program Verification

Set 2

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VTSA Summer School, Luxembourg, 2024

Part I

Dynamic Logic at Scale: The KeYsystem

KeY

KeY is an approach and tool for the

- Formal specification
- Deductive verification

of

- Software *source code*

using

- *Dynamic Logic*

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Versions of KeY support verification of

- sequential Java
- ABS (executable modelling language for distributed objects)
- Solidity (Smart Contracts, *work in progress*)

KeY Project Partners

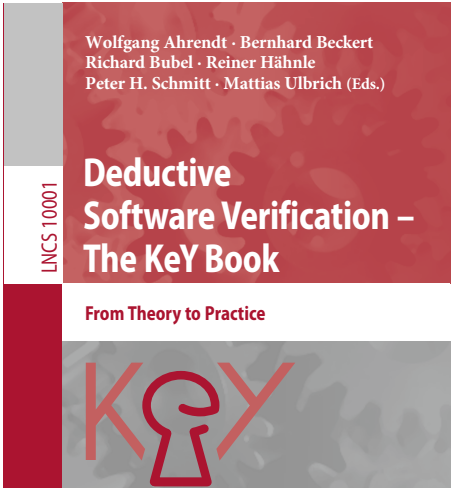
Karlsruhe Institute of Technology
Bernhard Beckert, Mattias Ulbrich, Peter H. Schmitt

Technical University Darmstadt
Reiner Hähnle, Richard Bubel

Chalmers University of Technology
Wolfgang Ahrendt

+ post-docs, PhD students, students, collaborators, alumni

The KeY Book


Wolfgang Ahrendt · Bernhard Beckert
Richard Bubel · Reiner Hähnle
Peter H. Schmitt · Mattias Ulbrich (Eds.)
LNCS 10001
Deductive
Software Verification –
The KeY Book
From Theory to Practice
KeY

KeY in 30 seconds

- Dynamic logic as program logic
- Calculus follows symbolic execution paradigm
- Sequent calculus
- Prover is interactive + automated
- most elaborate KeY instance: KeY-Java
  - Java as target language
  - Supports specification language JML (Java Modeling Language)

JML example

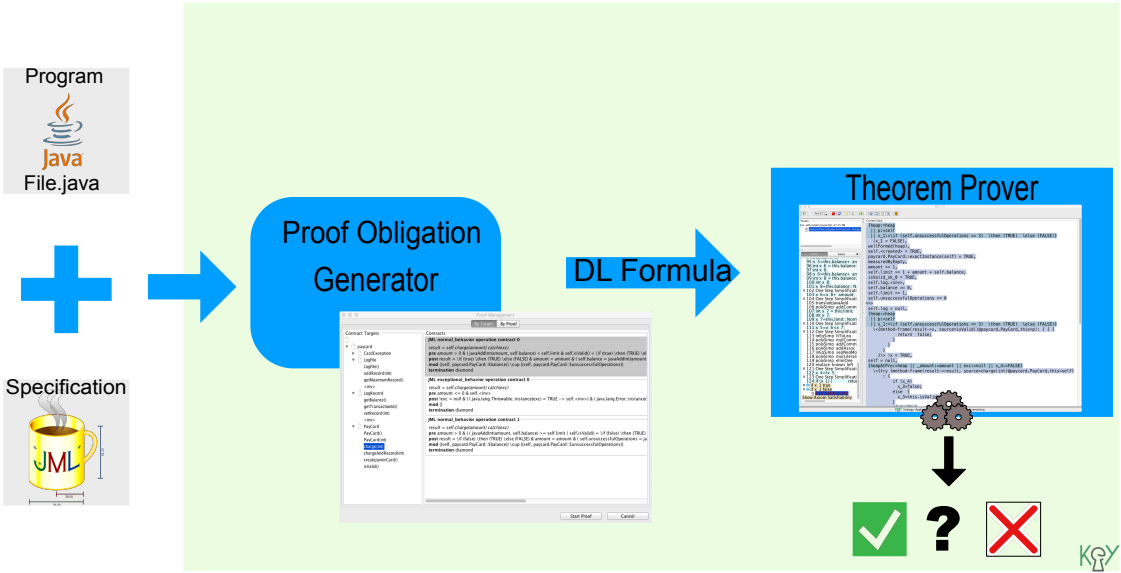
```
/*@ public normal_behavior
  @ requires a != null;
  @ ensures (\forall int j; j >= 0 && j < a.length;
  @                         \result >= a[j]);
  @ ensures a.length > 0 ==>
  @         (\exists int j; j >= 0 && j < a.length;
  @                         \result == a[j]);
  @*/
public static /*@ pure @*/ int max(int[] a) {
    int max = a[0], i = 1;
    while ( i < a.length ) {
      if ( a[i] > max ) max = a[i];
      ++i;
    }
    return max;}
```

JML to Dynamic Logic

Major components of KeY-Java

- Proof Obligation Generator
  - input: Java files containing JML specs
  - output: proof obligations in Dynamic Logic (DL) for Java
- KeY Prover
  - constructing proofs in sequent calculus for DL
  - designed for interplay of interaction and automated strategies

Typical KeY Workflow



Dynamic Logic for (almost) full Java

KeY supports full sequential Java, with some limitations:

- No generics
- No I/O
- No dynamic class loading or reflection
- API method calls: need either JML contract or implementation

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- Recently added: support for floating-point numbers/arithemic

Major Case Study with KeY: Timsort

Timsort

Hybrid sorting algorithm (insertion sort + merge sort) optimised for partially sorted arrays (typical for real-world data).

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Timsort is used in standard libraries of:

- Java
- Python
- Android
- Haskell [... and many more languages / frameworks]

Timsort: People

- Tim Peters

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- Sorting Algorithm Designer

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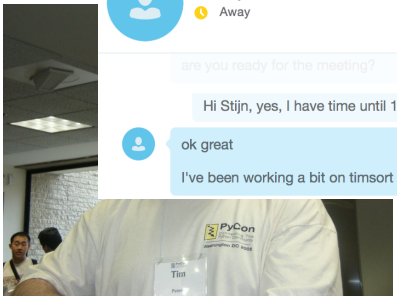

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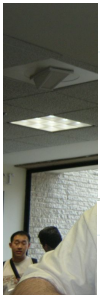

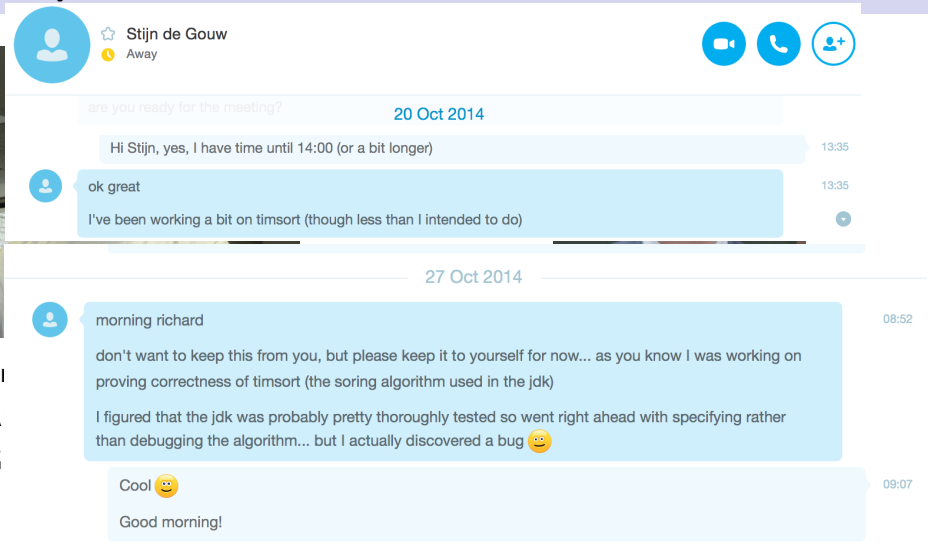

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- Python G

Assistant Professor

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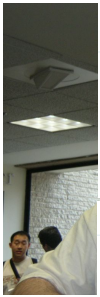

- Tim Peter
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Stijn de Gouw
Away

are you ready for the meeting?

20 Oct 2014

Hi Stijn, yes, I have time until 14:00 (or a bit longer) 13:35

ok great 13:35

I've been working a bit on timsort (though less than I intended to do)

27 Oct 2014

morning richard 08:52

don't want to keep this from you, but please keep it to yourself for now... as you know I was working on proving correctness of timsort (the soring algorithm used in the jdk)

I figured that the jdk was probably pretty thoroughly tested so went right ahead with specifying rather than debugging the algorithm... but I actually discovered a bug 😊

Cool 😊 09:07

Good morning!

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Found Bug in Java Libraries' main Sorting Method using KeY

- `java.util.Collections.sort` and `java.util.Arrays.sort` implement Timsort
- KeY verification of OpenJDK implementation revealed *bug*.

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Verified Fix using KeY

- Fixing the (OpenJDK) implementation
- Verified new version with KeY (no `IndexOutOfBoundsException`)

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- Fixing the (OpenJDK...
- Verified new version w... `ArrayIndexOutOfBoundsException`)

Some researchers found an error in the logic of merge_collapse, explained here, and with corrected code shown in ...

It should be fixed anyway, and their suggested fix looks good to me.

Tim Peters via Python-Bugtracker

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It should be fixed ... ...

Tim Peters via ...

Congratulations to Stijn de Gouw et al. for finding and fixing a bug in TimSort using formal methods!

Joshua Bloch via Twitter

...acker

Fixed and Verified the Implementation

Verification with KeY

- Adding support for bitwise operations to KeY
- 460 lines of specification vs. 928 lines of code
- Fixed version formally verified (absence of exceptions)
- Whole project 2.5 person month

Aftermath

- Java community choose sub-optimal fix (increasing stack size)

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  - Android
  - Python
  - Apache: Lucene, Hadoop, Spark++
  - Go, D, Haskell

Language	Min. array length req. to trigger error
Android	65.536 (2^{16})
Java	67.108.864 (2^{26})
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- blog with $>$3 million page views
- top news on ycombinator, reddit, Hacker News etc.

References

- Stijn de Gouw, Jurriaan Rot, Frank S. de Boer, Richard Bubel, Reiner Hähnle
  OpenJDK's Java.utils.Collection.sort() Is Broken: The Good, the Bad and the Worst Case
  CAV 2015

Part II

KeY Prover Intro: First-Order Sequent Calculus

Motivation for Introducing First-Order Logic

We verify JAVA programs using First-Order Dynamic Logic

First-order Java DL combines

- First-Order Logic (FOL)
- JAVA programs

First-Order Formulas

Formulas

- each atomic formula is a formula
- with φ and ψ formulas, x a variable, and τ a type, the following are also formulas:
  - $\neg\varphi$ *("not φ")*
  - $\varphi \wedge \psi$ *("φ and ψ")*
  - $\varphi \vee \psi$ *("φ or ψ")*
  - $\varphi \rightarrow \psi$ *("φ implies ψ")*
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Formulas with no free variable are 'closed'.

Reasoning by Syntactic Transformation

Prove validity of φ by **syntactic** transformation of φ

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Sequent Calculus based on notion of sequent:

$$\underbrace{\psi_1, \ldots, \psi_m}_{\text{antecedent}} \quad \Longrightarrow \quad \underbrace{\varphi_1, \ldots, \varphi_n}_{\text{succedent}}$$

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which (for closed formulas ψ_i, φ_i) is equivalent to

$$\{\psi_1, \ldots, \psi_m\} \quad \models \quad \varphi_1 \vee \cdots \vee \varphi_n$$

Notation for Sequents

$$\psi_1, \ldots, \psi_m \quad \Longrightarrow \quad \varphi_1, \ldots, \varphi_n$$

Consider antecedent/succedent as sets of formulas (possibly empty)

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Schema Variables

$\varphi, \psi, \ldots$ match formulas.
$\Gamma, \Delta, \ldots$ match sets of formulas.
Characterize infinitely many sequents with single schematic sequent, e.g.,

$$\Gamma \quad \Longrightarrow \quad \varphi \wedge \psi, \Delta$$

matches any sequent with occurrence of conjunction in succedent.

Here, we call $\varphi \wedge \psi$ main formula and Γ, Δ side formulas of sequent

Sequent Calculus Rules

Write syntactic transformation schema for sequents,
reflecting semantics of connectives

$$\text{RuleName}\ \frac{\overbrace{\Gamma_1 \Longrightarrow \Delta_1 \quad \cdots \quad \Gamma_r \Longrightarrow \Delta_r}^{\text{premisses}}}{\underbrace{\Gamma \Longrightarrow \Delta}_{\text{conclusion}}}$$

Meaning: For proving the conclusion, it suffices to prove all premisses.

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Admissible to have no premisses (then the rule is called 'axiom').

A rule is sound (correct) iff the validity of all premisses implies the validity of the conclusion.

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$$\text{close}\ \frac{}{\Gamma, \varphi \Longrightarrow \varphi, \Delta}$$

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close $\dfrac{}{\Gamma, \varphi \Longrightarrow \varphi, \Delta}$ true $\dfrac{}{\Gamma \Longrightarrow \mathrm{true}, \Delta}$ false $\dfrac{}{\Gamma, \mathrm{false} \Longrightarrow \Delta}$

	left side (antecedent)	right side (succedent)
not	$\dfrac{\Gamma \Longrightarrow \varphi, \Delta}{\Gamma, \neg\varphi \Longrightarrow \Delta}$	$\dfrac{\Gamma, \varphi \Longrightarrow \Delta}{\Gamma \Longrightarrow \neg\varphi, \Delta}$

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Sequent Calculus Proofs

Goal to prove: $\mathcal{G} = \quad \psi_1, \ldots, \psi_m \implies \varphi_1, \ldots, \varphi_n$

- find rule $\mathcal{R}$ whose conclusion matches $\mathcal{G}$
- instantiate $\mathcal{R}$ such that its conclusion is identical to $\mathcal{G}$
- apply that instantiation to all premisses of $\mathcal{R}$, resulting in new goals $\mathcal{G}_1, \ldots, \mathcal{G}_r$
- recursively find proofs for $\mathcal{G}_1, \ldots, \mathcal{G}_r$
- tree structure with goal as root
- close proof branch when applying rule without premiss

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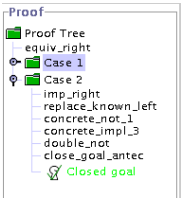
Goal-directed proof search

- Paper proofs: root at bottom, grow upwards
- KeY tool proofs: root at top, grow downwards



A Simple Proof

$$\dfrac{\dfrac{\dfrac{}{}\qquad\dfrac{}{}}{}}{\Longrightarrow (p \wedge (p \to q)) \to q}$$

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$$\dfrac{\dfrac{\dfrac{\dfrac{}{}\quad\dfrac{}{}}{p,\,(p \to q) \Longrightarrow q}}{p \wedge (p \to q) \Longrightarrow q}}{\Longrightarrow (p \wedge (p \to q)) \to q}$$

A Simple Proof

$$\dfrac{\dfrac{\dfrac{\dfrac{}{p \Longrightarrow p,\ q}\qquad\dfrac{}{p,\ q \Longrightarrow q}}{p,\ (p \to q) \Longrightarrow q}}{p \wedge (p \to q) \Longrightarrow q}}{\Longrightarrow (p \wedge (p \to q)) \to q}$$

A Simple Proof

$$\dfrac{\dfrac{\dfrac{\textsc{Close}\dfrac{*}{p \Longrightarrow p,\ q} \qquad \dfrac{*}{p,\ q \Longrightarrow q}\textsc{Close}}{p,\ (p \to q) \Longrightarrow q}}{p \wedge (p \to q) \Longrightarrow q}}{\Longrightarrow (p \wedge (p \to q)) \to q}$$

A Simple Proof

$$\dfrac{\dfrac{\dfrac{\dfrac{}{p \Longrightarrow p,\, q}\,{*}\,\textsc{Close} \qquad \dfrac{}{p,\, q \Longrightarrow q}\,{*}\,\textsc{Close}}{p,\, (p \to q) \Longrightarrow q}}{p \wedge (p \to q) \Longrightarrow q}}{\Longrightarrow (p \wedge (p \to q)) \to q}$$

A proof is closed iff all its branches are closed

Demo

`prop.key`

Proving Validity of First-Order Formulas

Proving a universally quantified formula

Claim: $\forall\, \tau\, x;\ \varphi$ is true

How is such a claim proved in Mathematics?

All even numbers are divisible by 2	$\forall\, \mathbf{int}\, x;\ (\mathsf{even}(x) \rightarrow \mathsf{divByTwo}(x))$
Let c be an arbitrary number	Declare "unused" constant $\mathbf{int}$ c
The even number c is divisible by 2	Prove $\mathsf{even}(c) \rightarrow \mathsf{divByTwo}(c)$

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Claim: $\forall\, \tau\, x;\ \varphi$ is true

How is such a claim proved in Mathematics?

All even numbers are divisible by 2	$\forall\, \mathbf{int}\, x;\ (\mathsf{even}(x) \rightarrow \mathsf{divByTwo}(x))$
Let c be an arbitrary number	Declare "unused" constant $\mathbf{int}$ c
The even number c is divisible by 2	Prove $\mathsf{even}(c) \rightarrow \mathsf{divByTwo}(c)$

Sequent rule $\forall$-right

$$\mathsf{forallRight}\ \frac{\Gamma \Longrightarrow [x/c]\,\varphi, \Delta}{\Gamma \Longrightarrow \forall\, \tau\, x;\ \varphi, \Delta}$$

- $[x/c]\,\varphi$ is result of replacing each occurrence of x in φ with c
- c **new** constant of type τ

Proving Validity of First-Order Formulas Cont'd

Proving an existentially quantified formula

Claim: $\exists\, \tau\, x;\ \varphi$ is true

How is such a claim proved in Mathematics?

There is at least one prime number	$\exists\, \mathbf{int}\, x;\ \mathrm{prime}(x)$
Provide any "witness", say, 7	Use variable-free term $\mathbf{int}\ 7$
7 is a prime number	Prove prime(7)

Proving Validity of First-Order Formulas Cont'd

Proving an existentially quantified formula

Claim: $\exists\, \tau\, x;\ \varphi$ is true

How is such a claim proved in Mathematics?

There is at least one prime number	$\exists\, \mathbf{int}\, x;\ \mathrm{prime}(x)$
Provide any "witness", say, 7	Use variable-free term $\mathbf{int}\ 7$
7 is a prime number	Prove prime(7)

Sequent rule $\exists$-right

$$\text{existsRight}\ \frac{\Gamma \Longrightarrow [x/t]\,\varphi,\ \exists\, \tau\, x;\ \varphi, \Delta}{\Gamma \Longrightarrow \exists\, \tau\, x;\ \varphi, \Delta}$$

- t any variable-free term of type τ
- We might need other instances besides t! Keep $\exists\, \tau\, x;\ \varphi$

Proving Validity of First-Order Formulas Cont'd

Using a universally quantified formula

We assume $\forall\, \tau\, x;\ \varphi$ is true.

How is such a fact used in a Mathematical proof?

We know that all primes are odd	$\forall\, \mathbf{int}\, x;\ (\mathsf{prime}(x) \rightarrow \mathsf{odd}(x))$
In particular, this holds for 17	Use variable-free term $\mathbf{int}\ 17$
We know: if 17 is prime it is odd	$\mathsf{prime}(17) \rightarrow \mathsf{odd}(17)$

Proving Validity of First-Order Formulas Cont'd

Using a universally quantified formula

We assume $\forall\, \tau\, x;\ \varphi$ is true.

How is such a fact used in a Mathematical proof?

We know that all primes are odd	$\forall\, \mathbf{int}\, x;\ (\mathsf{prime}(x) \to \mathsf{odd}(x))$
In particular, this holds for 17	Use variable-free term $\mathbf{int}\ 17$
We know: if 17 is prime it is odd	$\mathsf{prime}(17) \to \mathsf{odd}(17)$

Sequent rule $\forall$-left

$$\mathsf{forallLeft}\ \frac{\Gamma, \forall\, \tau\, x;\ \varphi,\ [x/t]\,\varphi \Longrightarrow \Delta}{\Gamma, \forall\, \tau\, x;\ \varphi \Longrightarrow \Delta}$$

- t any variable-free term of type τ
- We might need other instances besides t! Keep $\forall\, \tau\, x;\ \varphi$

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

We assume $\exists\, \tau\, x;\ \varphi$ is true

How is such a fact used in a Mathematical proof?

We know such an element exists. Let's give that element it a new name.

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

We assume $\exists\, \tau\, x;\ \varphi$ is true

How is such a fact used in a Mathematical proof?

We know such an element exists. Let's give that element it a new name.

Sequent rule $\exists$-left

$$\text{existsLeft}\ \frac{\Gamma, [x/c]\,\varphi \Longrightarrow \Delta}{\Gamma, \exists\, \tau\, x;\ \varphi \Longrightarrow \Delta}$$

- c **new** constant of type τ

Proving Validity of First-Order Formulas Cont'd

Using an equation between terms

We assume $t = t'$ is true

How is such a fact used in a Mathematical proof?

$x = y-1 \Longrightarrow (x+1)/y = 1$

Use $x = y-1$ to modify $(x+1)/y$:
Replace x in succedent with right-hand side of antecedent

Proving Validity of First-Order Formulas Cont'd

Using an equation between terms

We assume $t = t'$ is true

How is such a fact used in a Mathematical proof?

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Use $x = y-1$ to modify $(x+1)/y$:
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$x = y-1 \Longrightarrow (y-1+1)/y = 1$

Proving Validity of First-Order Formulas Cont'd

Using an equation between terms

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Replace x in succedent with right-hand side of antecedent

$x = y-1 \Longrightarrow (y-1+1)/y = 1$

Sequent rule =-left

$$\text{applyEqL}\ \frac{\Gamma, t = t', [t/t']\,\varphi \Longrightarrow \Delta}{\Gamma, t = t', \varphi \Longrightarrow \Delta} \qquad \text{applyEqR}\ \frac{\Gamma, t = t' \Longrightarrow [t/t']\,\varphi, \Delta}{\Gamma, t = t' \Longrightarrow \varphi, \Delta}$$

- Always replace left- with right-hand side (use eqSymm if necessary)
- t,t' variable-free terms of the same type

Proving Validity of First-Order Formulas Cont'd

Closing a subgoal in a proof

- We derived a sequent that is trivially valid

$$\text{close}\ \frac{}{\Gamma, \varphi \Longrightarrow \varphi, \Delta} \qquad \text{true}\ \frac{}{\Gamma \Longrightarrow \mathrm{true}, \Delta} \qquad \text{false}\ \frac{}{\Gamma, \mathrm{false} \Longrightarrow \Delta}$$

- We derived an equation that is trivially valid

$$\text{eqClose}\ \frac{}{\Gamma \Longrightarrow t = t, \Delta}$$

Sequent Calculus for FOL at One Glance

	left side, antecedent	right side, succedent
$\forall$	$\dfrac{\Gamma, \forall\, \tau\, x;\ \varphi,\ [x/t']\,\varphi \Longrightarrow \Delta}{\Gamma, \forall\, \tau\, x;\ \varphi \Longrightarrow \Delta}$	$\dfrac{\Gamma \Longrightarrow [x/c]\,\varphi, \Delta}{\Gamma \Longrightarrow \forall\, \tau\, x;\ \varphi, \Delta}$
$\exists$	$\dfrac{\Gamma, [x/c]\,\varphi \Longrightarrow \Delta}{\Gamma, \exists\, \tau\, x;\ \varphi \Longrightarrow \Delta}$	$\dfrac{\Gamma \Longrightarrow [x/t']\,\varphi,\ \exists\, \tau\, x;\ \varphi, \Delta}{\Gamma \Longrightarrow \exists\, \tau\, x;\ \varphi, \Delta}$
$=$	$\dfrac{\Gamma, t = t' \Longrightarrow [t/t']\,\varphi, \Delta}{\Gamma, t = t' \Longrightarrow \varphi, \Delta}$ (+ application rule on left side)	$\dfrac{}{\Gamma \Longrightarrow t = t, \Delta}$

- $[t/t']\,\varphi$ is result of replacing each occurrence of t in φ with t'
- t,t' variable-free terms of type τ
- c **new** constant of type τ (occurs not on current proof branch)
- Equations can be reversed by commutativity

Recap: 'Propositional' Sequent Calculus Rules

main	left side (antecedent)	right side (succedent)
not	$\dfrac{\Gamma \Longrightarrow \varphi, \Delta}{\Gamma, \neg\varphi \Longrightarrow \Delta}$	$\dfrac{\Gamma, \varphi \Longrightarrow \Delta}{\Gamma \Longrightarrow \neg\varphi, \Delta}$
and	$\dfrac{\Gamma, \varphi, \psi \Longrightarrow \Delta}{\Gamma, \varphi \wedge \psi \Longrightarrow \Delta}$	$\dfrac{\Gamma \Longrightarrow \varphi, \Delta \qquad \Gamma \Longrightarrow \psi, \Delta}{\Gamma \Longrightarrow \varphi \wedge \psi, \Delta}$
or	$\dfrac{\Gamma, \varphi \Longrightarrow \Delta \qquad \Gamma, \psi \Longrightarrow \Delta}{\Gamma, \varphi \vee \psi \Longrightarrow \Delta}$	$\dfrac{\Gamma \Longrightarrow \varphi, \psi, \Delta}{\Gamma \Longrightarrow \varphi \vee \psi, \Delta}$
imp	$\dfrac{\Gamma \Longrightarrow \varphi, \Delta \qquad \Gamma, \psi \Longrightarrow \Delta}{\Gamma, \varphi \to \psi \Longrightarrow \Delta}$	$\dfrac{\Gamma, \varphi \Longrightarrow \psi, \Delta}{\Gamma \Longrightarrow \varphi \to \psi, \Delta}$

close $\dfrac{}{\Gamma, \varphi \Longrightarrow \varphi, \Delta}$ true $\dfrac{}{\Gamma \Longrightarrow \mathrm{true}, \Delta}$ false $\dfrac{}{\Gamma, \mathrm{false} \Longrightarrow \Delta}$

Proving Validity of First-Order Formulas Cont'd

Example (A simple theorem about binary relations)

$$\exists x;\ \forall y;\ p(x,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)$$

"Untyped" logic: let type of x and y be *any*

Proving Validity of First-Order Formulas Cont'd

Example (A simple theorem about binary relations)

$$\frac{\forall y;\ p(c, y) \Longrightarrow \forall y;\ \exists x;\ p(x, y)}{\exists x;\ \forall y;\ p(x, y) \Longrightarrow \forall y;\ \exists x;\ p(x, y)}$$

$\exists$-left: substitute new constant c for x

Proving Validity of First-Order Formulas Cont'd

Example (A simple theorem about binary relations)

$$\dfrac{\dfrac{\forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}{\forall y;\ p(c,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}}{\exists x;\ \forall y;\ p(x,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}$$

$\forall$-right: substitute new constant d for y

Proving Validity of First-Order Formulas Cont'd

Example (A simple theorem about binary relations)

$$\dfrac{\dfrac{\dfrac{p(c,d),\ \forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}{\forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}}{\forall y;\ p(c,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}}{\exists x;\ \forall y;\ p(x,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}$$

$\forall$-left: free to substitute arbitrary term (of right type) for y, choose d

Proving Validity of First-Order Formulas Cont'd

Example (A simple theorem about binary relations)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{p(c,d),\ \forall y;\ p(c,y) \Longrightarrow p(c,d),\ \exists x;\ p(x,d)}{p(c,d),\ \forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}}{\forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}}{\forall y;\ p(c,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}}{\exists x;\ \forall y;\ p(x,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}$$

$\exists$-right: free to substitute arbitrary term (of right type) for x, choose c

Proving Validity of First-Order Formulas Cont'd

Example (A simple theorem about binary relations)

$$
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\dfrac{}{p(c,d),\ \forall y;\ p(c,y) \Longrightarrow p(c,d),\ \exists x;\ p(x,d)}\,*
}{p(c,d),\ \forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}
}{\forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}
}{\forall y;\ p(c,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}
}{\exists x;\ \forall y;\ p(x,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}
$$

Close

Proving Validity of First-Order Formulas Cont'd

Example (A simple theorem about binary relations)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\forall x;\ \exists y;\ p(x,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}\ *}{\forall y;\ p(c,y) \Longrightarrow \forall y;\ \exists x;\ p(x,y)}}{\forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}}{p(c,d),\ \forall y;\ p(c,y) \Longrightarrow \exists x;\ p(x,d)}}{p(c,d),\ \forall y;\ p(c,y) \Longrightarrow \exists x;\ p(c,d),\ \exists x;\ p(x,d)}$$

Demo

`relSimple.key`

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

Let x, y denote integer constants, both are not zero.

$$\neg(x = 0), \neg(y = 0) \qquad \Longrightarrow$$

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

Let x, y denote integer constants, both are not zero. We know further that x divides y.

$$\neg(x = 0), \neg(y = 0), \exists\, \texttt{int}\ k;\ y = k * x \Longrightarrow$$

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

Let x, y denote integer constants, both are not zero. We know further that x divides y.
Show: $(y/x) * x = y$
(equation not always true in integer division, e.g., $y = 1, x = 2$)

$$\neg(x = 0), \neg(y = 0), \exists\, \texttt{int}\ k;\ y = k * x \Longrightarrow (y/x) * x = y$$

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

Let x, y denote integer constants, both are not zero. We know further that x divides y.
Show: $(y/x) * x = y$
(equation not always true in integer division, e.g., $y = 1, x = 2$)
Proof: We know x divides y, i.e. there exists a k such that $y = k * x$. Let now c denote such a k.

$$\frac{\neg(x = 0), \neg(y = 0), y = c * x \Longrightarrow (y/x) * x = y}{\neg(x = 0), \neg(y = 0), \exists\, \texttt{int}\ k;\ y = k * x \Longrightarrow (y/x) * x = y}$$

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

Let x, y denote integer constants, both are not zero. We know further that x divides y.
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Proof: We know x divides y, i.e. there exists a k such that $y = k * x$. Let now c denote such a k. Hence we can replace y by $c * x$ on the right side.

$$\frac{\frac{\neg(x = 0), \neg(y = 0), y = c * x \Longrightarrow ((c * x)/x) * x = y}{\neg(x = 0), \neg(y = 0), y = c * x \Longrightarrow (y/x) * x = y}}{\neg(x = 0), \neg(y = 0), \exists\, \texttt{int}\ k;\ y = k * x \Longrightarrow (y/x) * x = y}$$

Proving Validity of First-Order Formulas Cont'd

Using an existentially quantified formula

Let x, y denote integer constants, both are not zero. We know further that x divides y.
Show: $(y/x) * x = y$
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Proof: We know x divides y, i.e. there exists a k such that $y = k * x$. Let now c denote such a k. Hence we can replace y by $c * x$ on the right side. ... $\square$

$$\dfrac{\dfrac{\dfrac{\dfrac{*}{\vdots}}{\neg(x = 0), \neg(y = 0), y = c * x \Longrightarrow ((c * x)/x) * x = y}}{\neg(x = 0), \neg(y = 0), y = c * x \Longrightarrow (y/x) * x = y}}{\neg(x = 0), \neg(y = 0), \exists\, \texttt{int}\ k;\ y = k * x \Longrightarrow (y/x) * x = y}$$

Features of the KeY Theorem Prover

Demo

`rel.key`, `twoInstances.key`

Feature List

- Can work on multiple proofs simultaneously (task list)
- Point-and-click navigation within proof
- Undo proof steps, prune proof trees
- Pop-up menu with proof rules applicable in pointer focus
- Preview of rule effect as tool tip
- Quantifier instantiation and equality rules by drag-and-drop
- Possible to hide (and unhide) parts of a sequent
- Saving and loading of proofs

Part III

Java Dynamic Logic

Java DL Example Formulas

$$(\texttt{balance} \geq c \land \texttt{amount} > 0) \rightarrow \langle \texttt{charge(amount);} \rangle \, \texttt{balance} > c$$

Java DL Example Formulas

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$$\texttt{o1.f} < \texttt{o2.f} \rightarrow [\texttt{t=o1.f; o1.f=o2.f; o2.f=t;}]\, \texttt{o2.f} < \texttt{o1.f}$$

Java DL Example Formulas

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$$\texttt{o1.f} > 0 \rightarrow [\texttt{o1.f=o1.f+o2.f; o2.f=o1.f-o2.f; o1.f=o1.f-o2.f;}]\, \texttt{o2.f} > 0$$

Java DL Example Formulas

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Last formula not valid: If $\texttt{o1} = \texttt{o2}$ in prestate, then $\texttt{o2.f} = 0$ in poststate.

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Last formula not valid: If $\texttt{o1} = \texttt{o2}$ in prestate, then $\texttt{o2.f} = 0$ in poststate.

Symbolic execution must take possible aliasing into account.

Java DL Example Formula

$$\forall\ \texttt{int}\ x;\ (\texttt{n} = x \land x >= 0 \rightarrow [\texttt{i = 0; r = 0; while (i<n) \{ i = i + 1; r = r + i;\} r=r+r-n;}]\ \texttt{r} = x * x)$$

Java DL Example Formula (in ASCII)

```
  a != null
->
  <
    int max = 0;
    if ( a.length > 0 ) max = a[0];
    int i = 1;
    while ( i < a.length ) {
      if ( a[i] > max ) max = a[i];
      ++i;
    }
  >
    \forall int j; (j >= 0 & j < a.length -> max >= a[j])
    &
    (a.length > 0 ->
      \exists int j; (j >= 0 & j < a.length & max = a[j]))
```

Assignment Rule Using Updates

Symbolic execution of assignment using updates

$$\text{assign}\ \frac{\Gamma \Longrightarrow \mathcal{U}\{x := t\}\langle \mathtt{rest}\rangle\varphi, \Delta}{\Gamma \Longrightarrow \mathcal{U}\langle x := t;\ \mathtt{rest}\rangle\varphi, \Delta}\quad t \text{ simple}$$

- $\mathcal{U}$: (nested) updates
- t simple: no side effects, no exceptions

Java DL Rules

Example rule

$$\text{if}\ \frac{\Gamma, \mathcal{U}(\texttt{b} = \mathit{true}) \Longrightarrow \mathcal{U}\langle \texttt{p};\ \mathit{rest}\rangle\varphi, \Delta \qquad \Gamma, \mathcal{U}(\texttt{b} = \mathit{false}) \Longrightarrow \mathcal{U}\langle \texttt{q};\ \mathit{rest}\rangle\varphi, \Delta}{\Gamma \Longrightarrow \mathcal{U}\langle \textbf{if}\ \texttt{(b) \{ p \} else \{ q \} ;}\ \mathit{rest}\rangle\varphi, \Delta}\ \text{b simple}$$

Symbolic execution must consider all possible execution branches

Java Dynamic Logic

Reasoning about Java programs requires extensions of FOL

- JAVA type hierarchy
- JAVA program variables
- JAVA heap for reference types

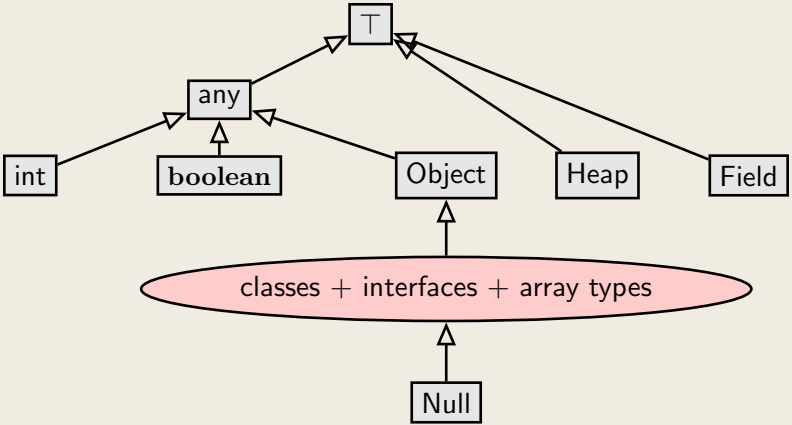
Type Hierarchy

Definition (Type Hierarchy)

- T_Σ is set of types
- Subtype relation $\sqsubseteq \subseteq T_\Sigma \times T_\Sigma$ with top element $\top$
  - $\tau \sqsubseteq \top$ for all $\tau \in T_\Sigma$

Modelling Java in FOL: Fixing a Type Hierarchy

T_Σ based on Java's type hierarchy (sketch)



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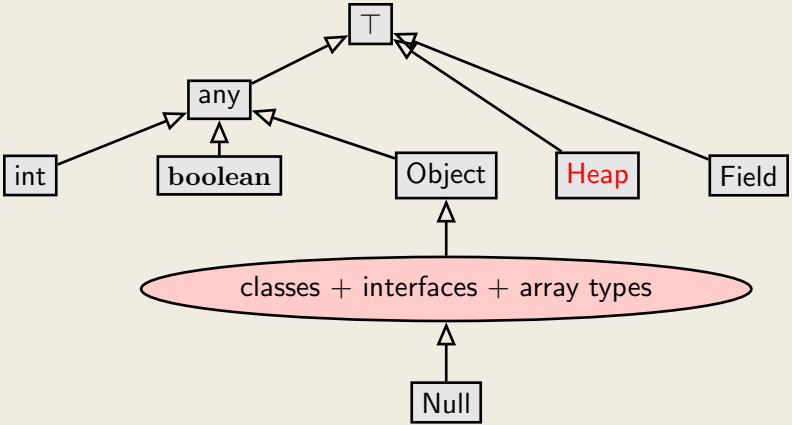

Modelling the Heap in FOL

The Java Heap

Objects are stored on (i.e., in) the heap.

- Status of heap changes during execution
- Each heap associates values to object/field pairs

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The Heap Model of KeY-DL

Each element of data type Heap represents a certain heap status.
Two functions involving heaps:

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The Heap Model of KeY-DL

Each element of data type Heap represents a certain heap status.
Two functions involving heaps:

- in F_Σ: Heap `store(Heap, Object, Field, any);`
  `store(`h, o, f, v`)` returns heap like h, but with v associated to $o.f$

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Two functions involving heaps:

- in F_Σ: Heap `store(Heap, Object, Field, any);`
  `store(`h, o, f, v`)` returns heap like h, but with v associated to $o.f$
- in F_Σ: any `select(Heap, Object, Field);`
  `select(`h, o, f`)` returns value associated to $o.f$ in h

Modelling the Heap in FOL

Modelling instance fields

Person
`int age` `int id`
`int setAge(int newAge)` `int getId()`

- for each JAVA reference type C there is a type $C \in T_\Sigma$, e.g., Person

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Reading Field id of Person p

FOL notation $\texttt{select}(h, \texttt{p}, \texttt{id})$

KeY notation $\texttt{p.id@}h$ (abbreviating $\texttt{select}(h, \texttt{p}, \texttt{id})$)

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KeY notation $\texttt{p.id@}h$ (abbreviating $\texttt{select}(h, \texttt{p}, \texttt{id})$)

$\texttt{p.id}$ (abbreviating $\texttt{select}(\texttt{heap}, \texttt{p}, \texttt{id})$)[a]

[a] heap is special program variable for "current" heap; mostly implicit in $o.f$

Modelling the Heap in FOL

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Writing to Field `id` of Person `p`

FOL notation $\texttt{store}(h, \texttt{p}, \texttt{id}, 6238)$

Modelling the Heap in FOL

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`int age` `int id`
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Writing to Field `id` of Person `p`

FOL notation $\texttt{store}(h, \texttt{p}, \texttt{id}, 6238)$

KeY notation $h[\texttt{p.id} := 6238]$ (notation for `store`, not update)

The Algebra of Heaps

We do *not* formalise the *structure* (implementation) of heaps.
We formalise the *behaviour*, with an algebra of heap operations:

$$\texttt{select}(\texttt{store}(h, o, f, v), o, f) =$$

The Algebra of Heaps

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Example

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$\quad \texttt{if } (\texttt{o} = \texttt{u}) \texttt{ then } 15 \texttt{ else } \texttt{select}(h, \texttt{u}, \texttt{f})$

Pretty Printing

Shorthand Notations for Heap Operations

$\texttt{o.f@h}$	is	$\texttt{select}(\texttt{h}, \texttt{o}, \texttt{f})$
$\texttt{h}[\texttt{o.f} := \texttt{v}]$	is	$\texttt{store}(\texttt{h}, \texttt{o}, \texttt{f}, \texttt{v})$

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Very-Shorthand Notations for Current Heap

Current heap always in special variable heap.

$\texttt{o.f}$	is	$\texttt{select(heap, o, f)}$
$\texttt{\{o.f := v\}}$	is update	$\texttt{\{heap := heap[o.f := v]\}}$

Field Update Assignment Rule

Changing the value of fields

How to (symbolically) execute assignment to field, e.g., `p.age=18;` ?

$$\text{assign}\ \frac{\Gamma \Longrightarrow \{\mathtt{o.f} := t\}\langle \mathtt{rest}\rangle\varphi, \Delta}{\Gamma \Longrightarrow \langle \mathtt{o.f = t;\ rest}\rangle\varphi, \Delta}$$

Admit on left-hand side of update JAVA location expressions

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But is this rule correct? See below.

Dynamic Logic: KeY input file

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Demo

`updates/firstAttributeExample.key`

Refined Semantics of Program Modalities

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Abrupt termination on top-level counts as non-termination!

Example Reconsidered: Exception Handling

```
\javaSource "path to source code";

\programVariables {
  ...
}

\problem {
       p != null -> \<{   p.age = 18;  }\> p.age = 18
}
```

Only provable when no top-level exception is thrown

Demo

`updates/secondAttributeExample.key`

Which Objects do Exist?

How to model object creation with **new** ?

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Constant Domain Assumption

Assume that domain $\mathcal{D}$ is the same in all states $(\mathcal{D}, \delta, \mathcal{I}) \in \mathit{States}$

Consequence:
Quantifiers and modalities commute:

$$\models (\forall\ T\ x;\ [\mathrm{p}]\varphi) \leftrightarrow [\mathrm{p}](\forall\ T\ x;\ \varphi)$$

Object Creation

Realizing Constant Domain Assumption

- Implicitly declared field **boolean** `<created>` in class `Object`
- `<created>` has value **true** iff argument object has been created
- Object creation modeled as $\{\texttt{heap} := \texttt{create(heap, ob)}\}$ for not (yet) created `ob` (essentially sets `<created>` field of `ob` to **true**)

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Alternatives exisit in the literature. E.g.:
[Ahrendt, de Boer, Grabe, *Abstract Object Creation in Dynamic Logic – To Be or Not To Be Created*, Springer, LNCS 5850]

Dynamic Logic to (almost) full Java

KeY supports full sequential Java, with some limitations:

- No concurrency
- No generics
- No I/O
- No dynamic class loading or reflection
- API method calls: need either JML contract or implementation

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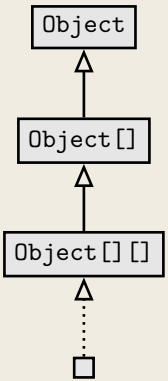
- Recently added: support for floating-point numbers/arithemic

Java Features in Dynamic Logic: Arrays

Arrays



- JAVA type hierarchy includes array types
- Types ordered according to JAVA subtyping rules
- Function `arr` : int $\to$ Field turns integer index into type Field (required in `store`).
- Store array elements on heap
- Value of `a[i]` in heap $\texttt{store}(\texttt{heap}, \texttt{a}, \texttt{arr}(\texttt{i}), 8)$ is 8
- Arrays `a` and `b` can refer to same object (aliasing)

Java Features in Dynamic Logic: Complex Expressions

Complex expressions with side effects

- JAVA expressions may have side effects, due to method calls, increment/decrement operators, nested assignments
- FOL terms have no side effect on the state

Example (Complex expression with side effects in Java)

```
int i = 0; if ((i=2)>= 2) i++;
```
value of `i` ?

Complex Expressions Cont'd

Decomposition of complex terms by symbolic execution

Follow the rules laid down in JAVA Language Specification

Local code transformations

$$\text{evalOrderIteratedAssgnmt}\ \frac{\Gamma \Longrightarrow \langle \texttt{y = t; x = y;}\ \omega\rangle\varphi, \Delta}{\Gamma \Longrightarrow \langle \texttt{x = y = t;}\ \omega\rangle\varphi, \Delta}\quad \texttt{t simple}$$

Temporary variables store result of evaluating subexpression

$$\text{ifEval}\ \frac{\Gamma \Longrightarrow \langle \textbf{boolean}\ \texttt{v0; v0 = b; if (v0) p;}\ \omega\rangle\varphi, \Delta}{\Gamma \Longrightarrow \langle \textbf{if}\ \texttt{(b) p;}\ \omega\rangle\varphi, \Delta}\quad \texttt{b complex}$$

Java Features in Dynamic Logic: Abrupt Termination

Abrupt Termination: Exceptions and Jumps

Redirection of control flow via **return**, **break**, **continue**, exceptions

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Demo

`exceptions/try-catch.key` (inspect file, auto-prove, inspect proof)

Java Features in Dynamic Logic: Null

Null pointer exceptions

There are no "exceptions" in FOL: interpretations $\mathcal{I}$ are total

Need to model possibility that $\mathsf{o} = \mathbf{null}$ in $\mathsf{o.a}$

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[a] Can be changed with Taclet Option runtimeExceptions

Field Update Assignment Rule Revisited (A)

Changing the value of fields

How to (symbolically) execute assignment to field?

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Revisit: Field Assignment Demo

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Demo

updates/secondAttributeExample.key

Java Features in Dynamic Logic: Aliasing

Demo

`aliasing/attributeAlias1.key`

Java Features in Dynamic Logic: Aliasing

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Reference Aliasing

Alias resolution causes proof split